UG Odd Semester (CBCS) Exam., December—2016

COMPUTER SCIENCE

(1st Semester)

Course No. : MCS–102 (C)

(Mathematics—I)

Full Marks : 75

Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define convergent sequence. Show that every convergent sequence in real line R converges to a unique point in R. 2+3=5

(b) Define monotonic sequence. If

$$x_n \quad \frac{1}{1\ 2} \quad \frac{1}{2\ 3} \quad \frac{1}{3\ 4} \qquad \frac{1}{n\ (n\quad 1)}$$

then show that (x_n) is a bounded monotonic sequence. 2+3=5

(c) State Cauchy criterion for convergence of sequence. By using Cauchy criterion, prove that the sequence (x_n) converges where

$$x_n \quad 1 \quad \frac{1}{2!} \quad \frac{1}{3!} \qquad \frac{1}{n!}$$ 2+3=5

2. *(a)* Define ratio test. By using ratio test, show that the following series converges : 2+3=5

$$\frac{1}{2} \quad \frac{2}{2^2} \quad \frac{3}{2^3} \qquad \frac{n}{2^n}$$

(b) Examine whether the series

$$2x \quad \frac{3x^2}{8} \quad \frac{4x^3}{27} \qquad \frac{(n\quad 1)}{n^3}\ x^n$$

is convergent or divergent for $(x\quad 0)$. 5

(c) Define limit comparison test, root test and Raabe's test for convergence of infinite series. 5

UNIT—II

3. *(a)* If a, b and c is a system of linearly independent vectors, then show that the following sets of vectors are also linearly independent : 3+3=6

(i) $a\quad b\quad c,\ b\quad c\quad a,\ 2a\quad 3b\quad 4c$

(ii) $2a\quad b\quad 3c,\ a\quad b\quad 2c,\ a\quad b\quad 3c$

(b) Find the area of the triangle whose vertices are the points with rectangular Cartesian coordinates as given below : 4

(1, 2, 3), (–2, 1, –4) and (3, 4, –2)

(c) Given two vectors, $\vec{a} \quad \hat{i} \quad \hat{j} \quad \hat{k}$ and $\vec{b} \quad \hat{i} \quad \hat{j} \quad \hat{k}$. Find a unit vector $\vec{c}$ perpendicular to the vector $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$. Find also a vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$. 5

4. *(a)* Find the angles between the following pairs of vectors : 2+2+2=6

(i) $\hat{i} \quad \hat{j} \quad \hat{k}, \quad \hat{i} \quad \hat{j} \quad 2\hat{k}$

(ii) $2\hat{i} \quad \hat{j} \quad \hat{k}, \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$

(iii) $3\hat{i} \quad 4\hat{j}, \quad 2\hat{j} \quad 5\hat{k}$

(b) Find the angle between the lines *AB* and *AC*, where *A*, *B*, *C* are three points with rectangular Cartesian coordinates

(1, 2, –1), (2, 0, 3) and (3, –1, 2) respectively. 5

(c) Compute the following vector product : 4

$$[(\hat{i} \quad \hat{j} \quad \hat{k}) \quad (2\hat{i} \quad 3\hat{j} \quad \hat{k})] \quad [(3\hat{i} \quad \hat{j} \quad \hat{k}) \quad (2\hat{j} \quad \hat{k})]$$

UNIT—III

5. *(a)* Show that if a circle $x^2 \quad y^2 \quad a^2$ cuts off by the straight line $y \quad mx \quad c$, a chord of length $2b$, then show that $c^2 \quad (1 \quad m^2)(a^2 \quad b^2)$. 5

(b) A circle touches the *x*-axis and cuts off an intercept of length $2a$ from *y*-axis. Prove that the locus of the centre of the circle is $x^2 \quad y^2 \quad a^2 \quad 0$. 5

(c) Show that the equation to the pair of straight lines through origin and perpendicular to $ax^2 \quad 2hxy \quad by^2 \quad 0$ is $bx^2 \quad 2hxy \quad ay^2 \quad 0$. 5

6. *(a)* Find the area of the triangle formed by the tangents drawn from $R(h, k)$ to the parabola $y^2 \quad 4ax$, and their chord of contact. 5

(b) Prove that the line joining two points on an ellipse such that the difference of their eccentric angles is constant, touches another ellipse. 5

(c) Show that the foot of the perpendicular drawn from the focus on any tangent to the hyperbola

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$$

lies on a circle. 5

UNIT—IV

7. *(a)* Solve the following system of equations by using Gauss-Jordan method : 5

$$\begin{array}{cccc} 2x & 5y & 3z & 9 \\ 3x & y & 2z & 3 \\ x & 2y & z & 6 \end{array}$$

(b) Solve the following system of equations by using triangularization method : 5

$$\begin{array}{cccc} x & y & z & 1 \\ 4x & 3y & z & 6 \\ 3x & 5y & 3z & 4 \end{array}$$

(c) Define equivalence of two matrices. Show that the following two matrices are equivalent with each other : 5

$$A \quad \begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 0 & 2 \end{matrix} \quad \text{and } B \quad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8. *(a)* Define eigenvalue and eigenvector of a matrix. Find all the eigenvalues and eigenvectors of the following matrix : 5

$$A \quad \begin{matrix} 1 & 3 & 3 \\ 0 & 5 & 6 \\ 0 & 3 & 4 \end{matrix}$$

(b) By using Cayley-Hamilton theorem, find the inverse of the following matrix : 5

$$A \quad \begin{matrix} 5 & 4 & 4 \\ 12 & 11 & 12 \\ 4 & 4 & 5 \end{matrix}$$

(c) Let ${}_1,\ {}_2,\ \ ,\ {}_k$ be distinct eigenvalues of a matrix A and $x_1,\ x_2,\ \ ,\ x_k$ be the corresponding eigenvectors of them respectively, then show that

$$\{x_1,\ x_2,\ \ ,\ x_k\}$$

is linearly independent. 5

UNIT—V

9. *(a)* Show that the following four matrices form a group under matrix multiplication : 5

$$A \ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix},\ B \ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix},\ C \ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix},\ \text{and } D \ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

(b) Show that the set of cube roots of unity $\{1,\ \ ,\ \ ^2\}$ forms a finite Abelian group with respect to multiplication. 5

(c) Define Abelian group. Let Q be the set of all rational numbers. Show that Q is an Abelian group with respect to the binary operation as defined below : 5

$$a \quad b \quad a \quad b \quad ab, \qquad a,b \quad Q$$

10. *(a)* Define commutative ring, division ring, zero divisor and field. 6

(b) Show that every finite integral domain is a field. 5

(c) Let S_7 is the set of integers under addition modulo 7 and multiplication modulo 7. Then show that S_7 is a ring. 4

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