2016/ODD/08/24/MCS–702 (C)/
MS–102 (C)/594

PG Odd Semester (CBCS) Exam., December—2016

COMPUTER SCIENCE

(7th Semester / 1st Semester)

Course No. : MCS–702 (C) / MS–102 (C)

(Design and Analysis of Algorithms)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Prove that the running time of an algorithm is $(g(n))$ iff its worst-case running time is $O(g(n))$ and its best-case running time is $(g(n))$. 3

(b) Show by substitution method that solution to $T(n)\ \ 2T(\ n/2\ )\ \ n$ is $O(n\lg n)$ and also $(n\lg n)$. Conclude that the solution is $(n\lg n)$. 6

(c) Solve the following by the Master method :

(i) $T(n)\ \ 2T(n/2)\ \ n\lg n$

(ii) $T(n)\ \ 7T(n/2)\ \ \ (n^2)$ 6

2. *(a)* Prove that if $\lim_{n} \frac{f(n)}{g(n)}\ \ 0$, then $f(n)\ \ O(g(n))$ but $g(n)\ \ O(f(n))$. 3

(b) Prove that if $\lim_{n} \frac{f(n)}{g(n)}$, then $f(n)\ \ \ (g(n))$ but $f(n)\ \ \ (g(n))$. 4

(c) For a given function $g(n, m)$ we denote by $O(g(n, m))$ the set of functions :

$O(g(n, m))\ \ \{f(n, m)$; there exist positive constants c, n_0 and m_0 such that $0\ \ f(n, m)\ \ c\,g(n,m)$ for all $n\ \ n_0$ or $m\ \ m_0\}$

Give the corresponding definitions for $(g(n, m))$ and $(g(n, m))$. 3

(d) Solve the following recurrence : 5

$$t_n \quad \begin{matrix} n, & \text{if } n\ \ 0 \text{ or } n\ \ 1 \\ 2t_{n\ \ 1}\ \ 2t_{n\ \ 2}, & \text{otherwise} \end{matrix}$$

UNIT—II

3. *(a)* Show that there are at most $\lceil n/2^{n \;\; 1} \rceil$ nodes of height h in any n-element heap. 5

(b) Show that an n-element heap has height $\lg n$. 4

(c) Illustrate the operation of HEAPSORT on the array

$$A \quad \langle 5,\ 13,\ 2,\ 25,\ 7,\ 17,\ 20,\ 8,\ 4 \rangle$$ 6

4. *(a)* Show that the running time of a selection algorithm to select the Kth largest/smallest element is linear in the worst case. 9

(b) In the algorithm of **4***(a)*, the input elements are divided into groups of 5. Will the algorithm work in linear time, if they are divided into groups of 7? Argue that the algorithm of **4***(a)* does not run in linear time, if groups of 3 are used. 6

UNIT—III

5. *(a)* Show how depth-first search works on the graph shown in Fig. 1 below. Assume that the DFS procedure considers the vertices in alphabetical order and each adjacency list is ordered alphabetically. Show the discovery and finishing times for each vertex and also show the classification of each edge. 6

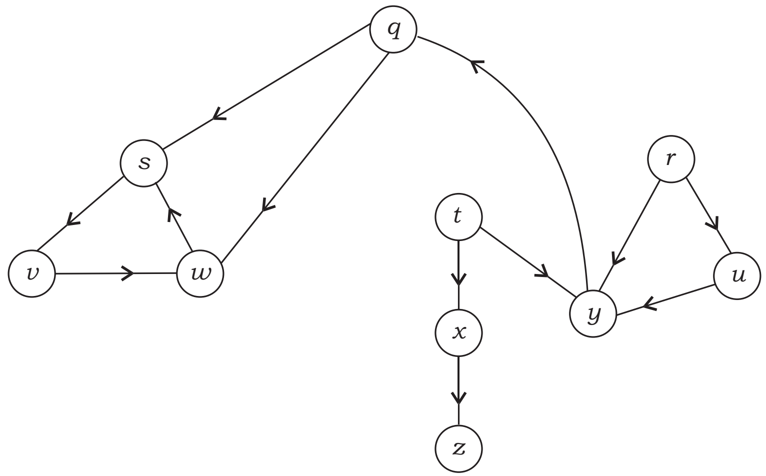

Fig. 1

(b) Compute a minimum cost spanning tree for the graph given in Fig. 2 using Prim's algorithm. 4

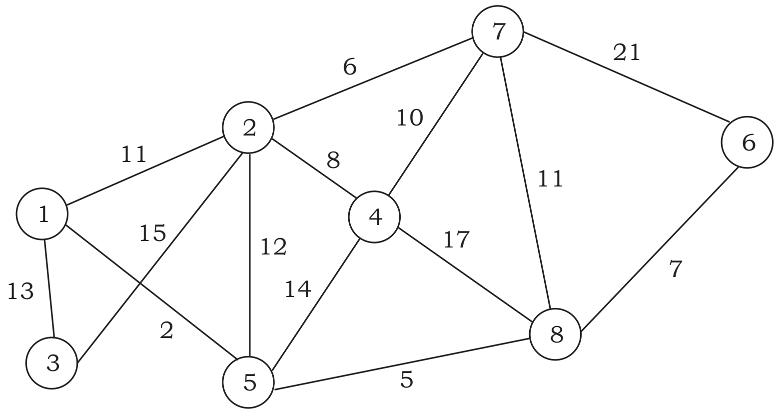

Fig. 2

(c) Show how the procedure STRONGLY-CONNECTED-COMPONENTS works on the graph of Fig. 1. Assume that DFS considers vertices in alphabetical order and that the adjacency list is ordered alphabetically. 5

6. *(a)* Run the Bellman-Ford algorithm on the directed graph of Fig. 3 below using vertex z as the source. Next, change the weight of edge (z, x) to 4 and run the algorithm again using s as the source. 3+3=6

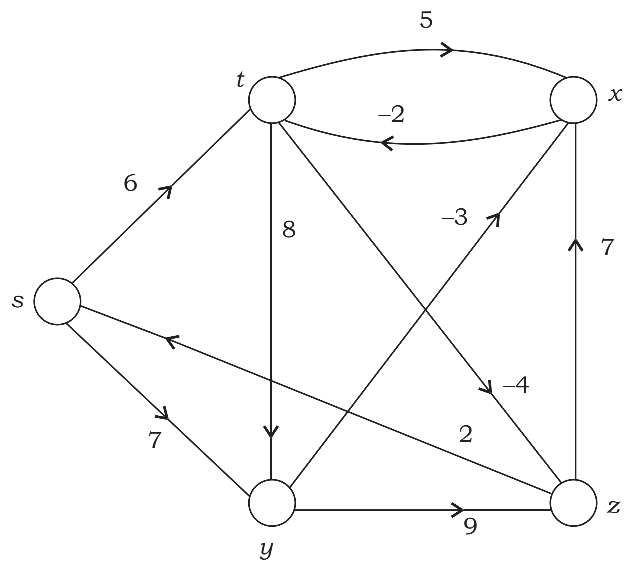

Fig. 3

(b) Why can negative weight and positive weight cycle not be present in a shortest path? 3

(c) Run Dijkstra's algorithm on the directed graph of Fig. 4 below using vertex z as the source. Show the d and values and the vertices in set s after each iteration. 6

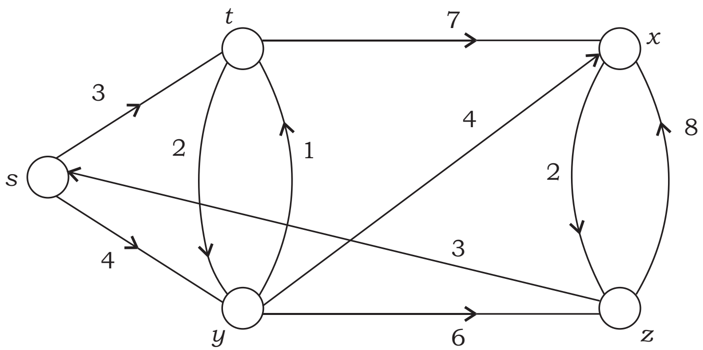

Fig. 4

Unit—IV

7. *(a)* Enumerate the sequence of steps needed to develop a dynamic programming algorithm. 3

(b) What is a memorized recursive algorithm? How is it more efficient than the conventional recursive procedure? 4

(c) Find the optimal parenthesization of a matrix chain product where the sequence of dimensions are

5 10, 10 3, 3 12, 12 5, 5 50, 50 6 8

8. *(a)* Define the following :

(i) Suffix function corresponding to a pattern P

(ii) Prefix function corresponding to a pattern P 5

(b) Draw a state transition diagram for a string matching automaton for the pattern

$a\,b\,a\,b\,b\,a\,b\,b\,a\,b\,a\,b\,b\,a\,b\,a\,b\,b\,a\,b\,b$

Over the alphabet $\{a,\ b\}$. 5

(c) Compute the prefix function for the pattern

$a\,b\,a\,b\,b\,a\,b\,b\,a\,b\,b\,a\,b\,a\,b\,b\,a\,b\,b$ 5

Unit—V

9. *(a)* Define the following : 6

(i) *NP*-complete problems

(ii) Reducibility

(b) Show that the satisfiatality of Boolean formulas in 3-conjunctive normal form is NP-complete. 9

10. *(a)* What are *P* and *NP* classes of problems? 5

(b) Show that the vertex cover problem is *NP*-complete. 10

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