UG Odd Semester (CBCS) Exam., December—2016

COMPUTER SCIENCE

(1st Semester)

Course No. : MCSCC–102

(Mathematics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* If A and B are two non-empty sets, then prove that

$$A \quad B \quad (A \quad B) \quad B$$ 5

(b) For any three non-empty sets A, B and C, prove that

$$A \quad (B \quad C) \quad (A \quad B) \quad (A \quad C)$$ 4

(c) By using Venn diagrams, prove the following identities : 5

(i) $A \quad (B \quad C) \quad (A \quad B) \quad (A \quad C)$

(ii) $A \quad (B \quad C) \quad (A \quad B) \quad (A \quad C)$

2. *(a)* Show that the map $f : Q \quad Q$, defined by $f(x) \quad 3x \quad 2$ is one-one and onto, where Q is the set of rational numbers. Also find $f^{\ \ 1}$. 4

(b) If the function $f : R \quad R$, be defined as $f(x) \quad 4x \quad 1$ and the function $g(x) \quad x^3 \quad 2$, then find $(g \circ f)(x)$ and $(f \circ g)(x)$. 5

(c) For an associative algebraic structure, show that the inverse of every element is unique. 5

UNIT—II

3. *(a)* Solve the following system of equations by using Gauss-Jordan method : 5

$$x \quad 5y \quad z \quad 14$$
$$2x \quad y \quad 3z \quad 13$$
$$3x \quad y \quad 4z \quad 17$$

(b) Solve the following system of equations by using Gauss elimination method : 4

$$x \quad y \quad z \quad 1$$
$$4x \quad 3y \quad z \quad 6$$
$$3x \quad 5y \quad 3z \quad 4$$

(c) Define equivalence of two matrices. Show that the following two matrices are equivalent with each other :

$$A \quad \begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 0 & 2 \end{array} \quad \text{and } B \quad \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$ 5

4. *(a)* Define eigenvalue and eigenvector of a matrix. Find all the eigenvalues and eigenvectors of the following matrix : 5

$$A \quad \begin{array}{ccc} 1 & 3 & 3 \\ 0 & 5 & 6 \\ 0 & 3 & 4 \end{array}$$

(b) By using Cayley-Hamilton theorem, find the inverse of the following matrix : 4

$$A \quad \begin{array}{ccc} 4 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{array}$$

(c) Define Hermitian, Skew-Hermitian, unitary and normal matrices with example. 5

UNIT—III

5. *(a)* Show that the three points A, B, C, with position vectors $2a \quad 3b \quad 5c$, $a \quad 2b \quad 3c$ and $7a \quad c$, respectively are collinear. Find the ratio in which C divides AB. Obtain also the equation of the line. 5

(b) Find the angle between the following pairs of vectors : 4

(i) $\hat{i} \quad \hat{j} \quad \hat{k}, \quad \hat{i} \quad \hat{j} \quad 2\hat{k}$

(ii) $2\hat{i} \quad \hat{j} \quad \hat{k}, \hat{i} \quad 2\hat{j} \quad \hat{k}$

(c) Determine a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$, where $\vec{a} \quad 4\hat{i} \quad 3\hat{j} \quad \hat{k}$ and $\vec{b} \quad 2\hat{i} \quad 6\hat{j} \quad 3\hat{k}$. Also obtain the sine of the angle between $\vec{a}$ and $\vec{b}$. 5

6. *(a)* Find the value of p so that the vectors $2\hat{i} \quad \hat{j} \quad \hat{k}$, $\hat{i} \quad 2\hat{j} \quad 3\hat{k}$ and $3\hat{i} \quad p\hat{j} \quad 5\hat{k}$ are coplanar. 4

(b) Compute the following vector products : 3+3=6

(i) $[(\hat{i} \quad \hat{j} \quad \hat{k}) \quad (2\hat{i} \quad 3\hat{j} \quad \hat{k})]$

$[(\quad 3\hat{i} \quad \hat{j} \quad \hat{k}) \quad (2\hat{j} \quad \hat{k})]$

(ii) $[(3\hat{i} \quad 2\hat{j} \quad 2\hat{k}) \quad (\hat{i} \quad \hat{k})]$

$[(\hat{i} \quad \hat{j} \quad \hat{k}) \quad (\hat{i} \quad 2\hat{j} \quad 3\hat{k})]$

(c) Define gradient of a scalar point function $f(x, y, z)$. If

$$f(x, y, z) \quad 3x^2y \quad y^3z \quad xyz$$

then find gradient of f at the point $(1, \quad 2, \quad 1)$. 4

UNIT—IV

7. *(a)* Verify that the following equations are homogeneous and hence solve them : 2+2=4

(i) $(x^2 \quad 2y^2)\,dx \quad xy\,dy \quad 0$

(ii) $x\dfrac{dy}{dx} \quad 2x \quad 3y$

(b) Show that the following equations are exact and hence solve them : 5

(i) $\cos x \cos^2 y\,dx \quad 2\sin x \sin y \cos y\,dy \quad 0$

(ii) $(\sin x \tan y \quad 1)\,dx \quad \cos x \sec^2 y\,dy \quad 0$

(c) Solve the following linear equations : 5

(i) $(1 \quad x^2)\,dy \quad 2xy\,dx \quad \cot x\,dx$

(ii) $y \quad x \quad xy\cot x \quad xy \quad 0$

8. *(a)* Show that $y \quad c_1 \sin x \quad c_2 \cos x$, is the general solution of $y \quad y \quad 0$ and also find the particular solution for which $y(0) \quad 2$ and $y(0) \quad 3$. 4

(b) Solve the following initial value problems : 5

(i) $y \quad 6y \quad 9y \quad 0,\ y(0) \quad 0$ and $y(0) \quad 5$

(ii) $y \quad 4y \quad 2y \quad 0,\ y(0) \quad 1$ and $y(0) \quad 2 \quad 3\sqrt{2}$

(c) By using the method of undetermined coefficients, solve the following differential equation : 5

$$y \quad 2y \quad 5y \quad 25x^2 \quad 12$$

UNIT—V

9. *(a)* Find the Laplace transform of the following : 3+3=6

(i) $\dfrac{1 \quad e^t}{t}$

(ii) $\dfrac{\cos 2t \quad \sin 3t}{t}$

(b) Find the Laplace transform of

$$f(t) \quad \begin{matrix} t \quad , & 0 \quad t \quad 6 \\ 12 \quad t, & 6 \quad t \quad 12 \end{matrix}$$

given that $f(t \quad 12) \quad f(t)$, for $t \quad 0$. 4

(c) Obtain the inverse Laplace transform of

$$\frac{4s^2 \quad 3s \quad 5}{(s \quad 1)(s^2 \quad 2s \quad 2)}$$ 4

10. *(a)* By using shifting property, find the inverse Laplace transform of the following : 2+3+3=8

(i) $\dfrac{1}{(s \quad a)^n}$

(ii) $\dfrac{s \quad 2}{s^2 \quad 4s \quad 13}$

(iii) $\dfrac{2s \quad 3}{s^2 \quad s \quad 3/4}$

(b) Solve the following differential equation by using Laplace transform method :

$$y \quad 4y \quad 8y \quad \cos 2t$$

given that $y \quad 2$ at $y \quad 1$, when $t \quad 0$. 4

(c) State the convolution theorem for Laplace transform. 2

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