UG Odd Semester (CBCS) Exam., December—2016

COMPUTER SCIENCE

(3rd Semester)

Course No. : MCSCC–302

(Mathematics—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Let $f(x)$ and (x) be any two real valued functions and if $\underset{x \quad a}{\text{Lt}} f(x) \quad l$, and $\underset{x \quad a}{\text{Lt}} (x) \quad l$, where l and l are finite real numbers, then show that—

(i) $\underset{x \quad a}{\text{Lt}} \{f(x) \quad (x)\} \quad l \quad l$;

(ii) $\underset{x \quad a}{\text{Lt}} \{f(x) \quad (x)\} \quad ll$. 5

(b) By using definition of limit, show that

$$\underset{x \quad 1}{\text{Lt}} \frac{x^2 \quad x \quad 1}{x \quad 1} \quad \frac{1}{2}$$ 5

(c) Show that

$$\underset{x \quad 0}{\text{Lt}} \frac{\sin x}{x} \quad 1$$ 4

2 *(a)* Discuss the continuity of the following functions at the points indicated : 2½+2½=5

(i) $$f(x) \quad \begin{cases} x, & \text{if } 0 \quad x \quad 1 \\ 2 \quad x, & \text{if } 1 \quad x \quad 2 \\ x \quad \frac{1}{2}x^2, & \text{if } \quad x \quad 2 \end{cases}$$

at the point $x \quad 2$

(ii) $$f(x) \quad \begin{cases} x^2 \quad x, & \text{if } 0 \quad x \quad 1 \\ 2, & \text{if } \quad x \quad 1 \\ 2x^3 \quad x \quad 1, & \text{if } 1 \quad x \quad 2 \end{cases}$$

at the point $x \quad 1$

(b) State and prove Cauchy's mean-value theorem. 5

(c) Evaluate the following limits : 2+2=4

(i) $$\underset{x \quad 1}{\text{Lt}} \frac{x^2 \quad 3x \quad 2}{x^2 \quad 4x \quad 3}$$

(ii) $$\underset{x \quad 0}{\text{Lt}} \frac{\sqrt{1 \quad 2x} \quad \sqrt{1 \quad 3x}}{x}$$

UNIT—II

3. *(a)* By using the definition of derivative, prove that—

(i) $\frac{d}{dx}(\cos x) \quad \sin x$;

(ii) $\frac{d}{dx}(\sin x) \quad \cos x.$ 5

(b) If (x) and (x) are differentiable functions, then by using the definition of derivative, show that

$$\frac{d}{dx} \frac{(x)}{(x)} \quad \frac{(x) \quad (x) \quad (x) \quad (x)}{\{ \quad (x)\}^2}$$ 5

(c) If $y \quad \frac{\sqrt{1 \quad x^2} \quad 1}{x}$, then find $\frac{dy}{dx}$. 4

4. *(a)* Discuss the differentiability of the following functions at the points indicated : 3+3=6

(i) $f(x) \quad \begin{array}{ll} x^2 \quad x \quad 1, & \text{if } 0 \quad x \quad 1 \\ 2x \quad 1, & \text{if } 1 \quad x \quad 2 \end{array}$

at the point $x \quad 1$

(ii) $f(x) \quad \begin{array}{ll} x^2 \quad x, & \text{if } 0 \quad x \quad 1 \\ 2, & \text{if } \quad x \quad 1 \\ 2x^3 \quad x \quad 1, & \text{if } 1 \quad x \quad 2 \end{array}$

at the point $x \quad 1$

(b) If $y \quad x \quad \cfrac{1}{x \quad \cfrac{1}{x \quad \cfrac{1}{x \quad \cfrac{1}{x \quad \text{to}}}}}$

then show that

$$\frac{dy}{dx} \quad \cfrac{1}{2 \quad \cfrac{x}{x \quad \cfrac{1}{x \quad \cfrac{1}{x \quad \text{to}}}}}$$ 4

(c) If $f(a) \quad 2, \; f\,(a) \quad 1, \; g(a) \quad 1, \; g\,(a) \quad 2,$ then find the value of

$$\underset{x \quad a}{\text{Lt}} \frac{g(x) \quad f(a) \quad g(a) \quad f(x)}{x \quad a}$$ 4

UNIT—III

5. *(a)* By using L'Hospital's rule find the following limits : 4+4=8

(i) $\underset{x \quad 0}{\text{Lt}} \left(\frac{\tan x}{x}\right)^{1/x}$

(ii) $\underset{x \quad 0}{\text{Lt}} \left(\frac{\sin x}{x}\right)^{1/x^2}$

(b) If $y \quad a\cos(\log x) \quad b\sin(\log x)$, then prove that

$$x^2 y_2 \quad xy_1 \quad y \quad 0$$

and $x^2 y_{n \quad 2} \quad (2n \quad 1)\, xy_{n \quad 1} \quad (n^2 \quad 1)\, y_n \quad 0$

3+3=6

6. *(a)* Find the extreme values of the following functions if they exist : 5

(i) $f(x) \quad 8x^5 \quad 15x^4 \quad 10x^3$

(ii) $f(x) \quad 4x^3 \quad 15x^2 \quad 12x \quad 2$

(b) If $u \quad \frac{y}{z} \quad \frac{z}{x} \quad \frac{x}{y}$, then prove that

$$x\frac{u}{x} \quad y\frac{u}{y} \quad z\frac{u}{z} \quad 0$$ 4

(c) If

$$f(x, y) \quad \begin{cases} xy\dfrac{x^2 \quad y^2}{x^2 \quad y^2}, & \text{when } x \quad 0 \text{ or } y \quad 0 \\ 0, & \text{when } x \quad 0 \text{ and } y \quad 0 \end{cases}$$

then show that at the point (0, 0)

$$\frac{{}^2 f}{x\ y} \quad \frac{{}^2 f}{y\ x}$$ 5

UNIT—IV

7. *(a)* If $X \quad (x_n)$ is a sequence of real numbers converges to x and $Z \quad (z_n)$ is a sequence of non-zero real numbers that converges to z and if $z \quad 0$, then show that the sequence $X / Z \quad (x_n / z_n)$ converges to x / z. 5

(b) Show that every convergent sequence of real numbers is bounded. Is every bounded sequence convergent? Justify. 5

(c) Define Cauchy sequence. If $X \quad (x_n)$ is a convergent sequence of real numbers, then show that (x_n) is a Cauchy sequence. 4

8. *(a)* Show that the infinite series

$$1 \quad \frac{1}{2^p} \quad \frac{1}{3^p} \quad \frac{1}{4^p}$$

is convergent if $p \quad 1$. 6

(b) Examine whether the following series is convergent or divergent : 5

$$x \quad \frac{1}{2}\frac{x^3}{3} \quad \frac{1\ 3}{2\ 4}\frac{x^5}{5} \quad \frac{1\ 3\ 5}{2\ 4\ 6}\frac{x^7}{7}$$

(c) Test the convergence of the following series : 3

$$\frac{1}{3} \quad \frac{2}{5}^{\ 2} \quad \frac{3}{7}^{\ 3} \quad \frac{4}{9}^{\ 4}$$

UNIT—V

9. *(a)* Define Riemann integration. If f is a Riemann integrable function, then show that the value of the Riemann integration is unique. 5

(b) If f and g are Riemann integrable functions on the closed interval $[a, b]$, then show that

$$\int_a^b (f \quad g)\, dx \quad \int_a^b f dx \quad \int_a^b g dx$$ 5

(c) Define step function. If $:[a, b] \quad R$ is a step function, then show that is Riemann integrable. 4

10. *(a)* Evaluate the double integral

$$\int_E e^{(x \quad y)} dxdy$$

where E is the domain, which lies between two squares of sides 2 and 4, with centre at the origin and sides parallel to the axes. 5

(b) Evaluate $(x^2 \quad y^2)\, dxdy$, over the domain bounded by $xy \quad 1$, $y \quad 0$, $y \quad x$ and $x \quad 2$. 4

(c) Evaluate $\int_R f(x, y)\, dxdy$, over the rectangle $R \quad [0, 1; 0, 1]$, where

$$f(x, y) \quad \begin{cases} x \quad y, & \text{if } x^2 \quad y \quad 2x^2 \\ 0, & \text{otherwise} \end{cases}$$ 5

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