UG Odd Semester (CBCS) Exam., December—2016

COMPUTER SCIENCE

(3rd Semester)

Course No. : MCS–302 (C)

(Mathematics—II)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Let $f(x)$ and (x) be any two real valued functions and if

$$\lim_{x \to a} f(x) = l$$

and

$$\lim_{x \to a} (x) = l$$

where l and l are finite real numbers, then show that

(i) $\lim_{x \to a} \{f(x) \quad (x)\} = l \quad l$

(ii) $\lim_{x \to a} \{f(x) \quad (x)\} = l\, l$ 5

(b) Using definition of limit, prove that

$$\lim_{x \to 2} \frac{x^3 \quad 4}{x^2 \quad 1} = \frac{4}{5}$$ 5

(c) Evaluate the following limits : 5

(i) $\lim_{x \to 0} \frac{x^2 \sin(\frac{1}{x})}{\sin x}$

(ii) $\lim_{x \to 0} \frac{\sin x \quad \tan x}{x^3}$

2. *(a)* Discuss the continuity of the following functions at the points indicated : 3+3=6

(i) $f(x) = \begin{cases} x, & \text{if } 0 < x < 1 \\ 2 \quad x, & \text{if } 1 \quad x \quad 2 \\ x \quad \frac{1}{2}x^2, & \text{if } \quad x \quad 2 \end{cases}$

at the point $x \quad 2$.

(ii) $f(x) = \begin{cases} x^2 \quad x, & 0 \quad x < 1 \\ 2, & x \quad 1 \\ 2x^3 \quad x \quad 1, & 1 \quad x \quad 2 \end{cases}$

at the point $x \quad 1$.

(b) State and prove Cauchy mean-value theorem. 6

(c) Show that $f(x) \quad \log(x \quad \sqrt{1 \quad x^2})$ is an odd function. 3

UNIT—II

3. *(a)* By using the definition of derivative, prove the following equality : 5

(i) $\frac{d}{dx}(\cos x) \quad \sin x$

(ii) $\frac{d}{dx}(\sin x) \quad \cos x$

(b) If (x) and (x) are differentiable functions, then by using the definition of derivative, show that

$$\frac{d}{dx}\frac{(x)}{(x)} \quad \frac{(x)\ (x) \quad (x)\ (x)}{\{\ (x)\}^2}$$ 5

(c) If

$$y \quad \frac{\sqrt{1 \quad x^2} \quad 1}{x}$$

find $\frac{dy}{dx}$. 5

4. *(a)* A function $f(x)$ is defined in $0 \quad x \quad 2$, by

$$f(x) \quad \begin{matrix} x^2 \quad x \quad 1, & 0 \quad x \quad 1 \\ 2x \quad 1\ , & 1 \quad x \quad 2 \end{matrix}$$

Examine the continuity and differentiability of $f(x)$ at $x \quad 1$. 5

(b) If $y \quad a\cos(\log x) \quad b\sin(\log x)$, then prove that

$$x^2 y_2 \quad x y_1 \quad y \quad 0$$

and

$$x^2 y_{n\ 2} \quad (2n \quad 1)\, x\, y_{n\ 1} \quad (n^2 \quad 1)\, y_n \quad 0$$ 5

(c) Find the extreme values of the following functions, if they exist : 5

(i) $f(x) \quad 8x^5 \quad 15x^4 \quad 10x^3$

(ii) $f(x) \quad 4x^3 \quad 15x^2 \quad 12x \quad 2$

UNIT—III

5. *(a)* Define Riemann integration. If f is a Riemann integrable function, then show that the value of the Riemann integration is unique. 2+4=6

(b) If f and g are Riemann integrable functions on the closed interval $[a, b]$, then show that

$$\int_a^b (f \quad g)\, dx \quad \int_a^b f dx \quad \int_a^b g dx$$ 5

(c) Define step function. If $: [a, b] \quad R$, is a step function, then show that is Riemann integrable. 4

6. *(a)* Evaluate the double integral

$$\iint_E e^{(x\ \ y)}\,dx\,dy$$

where E is the domain, which lies between two squares of sides 2 and 4, with centre at the origin and sides parallel to the axes. 5

(b) Evaluate

$$\iint (x^2\ \ y^2)\,dx\,dy$$

over the domain bounded by, $xy\ \ 1$, $y\ \ 0$, $y\ \ x$ and $x\ \ 2$. 4

(c) Evaluate

$$\iint_R f(x, y)\,dx\,dy$$

over the rectangle, $R\ \ [0, 1; 0, 1]$, where

$$f(x, y)\ \ \begin{array}{ll} x\ \ y, & \text{if}\quad x^2\ \ y\ \ 2x^2 \\ 0\ , & \text{elsewhere}\end{array}$$ 6

UNIT—IV

7. *(a)* Verify that the following equations are homogeneous and hence solve them : 2½+2½=5

(i) $x^2y\ \ 3xy\ \ 2y^2\ \ 0$

(ii) $xy\ \ 2x\ \ 3y$

(b) Check whether the following differential equations are exact or not. If exact, also solve by the method of exactness : 3+2=5

(i) $\cos x\cos^2 y\,dx\ \ 2\sin x\sin y\cos y\,dy\ \ 0$

(ii) $x\ \ \dfrac{2}{y}\,dy\ \ y\,dx\ \ 0$

(c) Solve the following linear equations : 2½+2½=5

(i) $y\ \ x\ \ xy\cot x\ \ xy\ \ 0$

(ii) $y\ \ y\cot x\ \ 2x\,\mathrm{cosec}\,x$

8. *(a)* Show that $y\ \ c_1\sin x\ \ c_2\cos x$, is the general solution of $y\ \ y\ \ 0$, on any interval, and also find the particular solution for which $y(0)\ \ 2$ and $y'(0)\ \ 3$. 5

(b) Find the solutions of the following initial-value problems : 2½+2½=5

(i) $y\ \ 8y\ \ 9y\ \ 0,\ y(1)\ \ 2,\ y'(1)\ \ 0$

(ii) $y\ \ 5y\ \ 6y\ \ 0,\ y(1)\ \ e^2,\ y'(1)\ \ 3e^2$

(c) By using the method of undetermined coefficients, solve the following equation : 5

$$y\ \ 3y\ \ 2y\ \ 14\sin 2x\ \ 18\cos 2x$$

UNIT—V

9. *(a)* Find the Fourier series of the function defined by

$$f(x) \quad \begin{matrix} 0, & \text{if} & x & 0 \\ , & \text{if} & 0 & x \end{matrix}$$

5

(b) Find the sine series and also the cosine series for the function

$f(x) \quad \sin x, \quad 0 \quad x$ 3+3=6

(c) Define Fourier sine series and cosine series of a function $f(x)$. 2+2=4

10. *(a)* Find the Fourier transform of $f(x) \quad e^{\ a|x|}, \qquad x \quad .$ 5

(b) If $F(s)$ is the Fourier transform of $f(x)$, then show that the Fourier transform of $f(x).\cos ax$ is

$$\frac{1}{2}[F(s \quad a) \quad F(s \quad a)]$$

5

(c) State and prove shifting property in Fourier transform. 5

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