PG Odd Semester (CBCS) Exam., December—2016

PHYSICS

(1st Semester)

Course No. : PH–102 (C)

(Mathematical Physics—I)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define vector space. What do you mean by inner product of two vectors? 2+1=3

(b) Prove Schwarz inequality

$$|(u, v)|^2 \quad (u,u)(v,v)$$ 6

(c) Define basis in a vector space. Discuss the Gram-Schmidt orthonormalization process. 1+5=6

2. *(a)* Define Hermitian operator. Show that if H is a Hermitian operator, then $U \quad e^{iH}$ is unitary. 2+3=5

(b) Consider the matrix

$$A \begin{matrix} \cos & \sin \\ \sin & \cos \end{matrix}$$

where is a real number.

(i) Show that it is an orthogonal matrix. 3

(ii) Is it a unitary matrix? 1

(iii) Obtain its eigenvalues and eigenvectors. 3+3=6

UNIT—II

3. *(a)* Define analytic function and deduce the Cauchy-Riemann conditions. 1+4=5

(b) State and prove Cauchy's integral formula. 2+5=7

(c) Evaluate

$$\oint_C \frac{\sin z^2 \quad \cos z^2}{(z \quad 1)(z \quad 2)} dz$$

where C is the circle $|z|$ 3. 3

4. *(a)* State and prove Cauchy's residue theorem. 2+4=6

(b) Evaluate

$$\frac{1}{2\ i}\oint_C \frac{e^{zt}}{z^2(z^2\ \ 2z\ \ 2)}dz$$

around the circle C with equation $|z|\ \ 3$. 6

(c) Find the residues of

$$f(z)\ \ \frac{z^2\ \ 2z}{(z\ \ 1)^2(z^2\ \ 4)}$$ 3

UNIT—III

5. *(a)* Define group. When are two elements of a group said to commute? What is the order of a group? 2+1+1=4

(b) When a group is called *(i)* Abelian, *(ii)* finite and *(iii)* infinite? 3

(c) Show that if $a\ \ G$ has finite order $O(a)$, then the exponents of a are precisely the integral multiples of $O(a)$. 2

(d) Prove Schur's lemma. 6

6. *(a)* Define homomorphism, isomorphism and automorphism. 6

(b) Show that $\sum_i n_i^2\ \ g$, where g is the order of the group and n_i is the dimension of ith irreducible representation of the group and the summation over i runs over all the irreducible representation of the group. 9

UNIT—IV

7. *(a)* When a differential equation is called as partial? What is the order of a partial differential equation? Define linear and nonlinear partial differential equations. 1+1+2=4

(b) Solve the equation

$$\frac{q}{y}\ \ xq\ \ x^2$$ 5

(c) Explain the differences between node and saddle point. 4

(d) Write Laplace's equation in spherical polar coordinate. 2

8. *(a)* Discuss the different trajectories obtained in $\dot{}$, $\frac{d}{dt}$ plane for the motion of a simple pendulum

$$\frac{d^2}{dt^2} \quad \frac{g}{l}\sin$$

6

(b) Classify the nature of critical points for the equations

$$\dot{x} \quad ax \quad by$$
$$\dot{y} \quad cx \quad dy$$

9

UNIT—V

9. *(a)* Express the velocity $\bar{v}$ $\frac{d\bar{r}}{dt}$ and the acceleration $\bar{a}$ $\frac{d^2\bar{r}}{dt^2}$ in cylindrical coordinates.

8

(b) State and prove Gauss' theorem. 7

10. *(a)* Define contravariant, covariant and mixed tensors. 3

(b) Show that Kronecker delta is a mixed tensor of rank 2. 4

(c) Define metric tensor and covariant derivative of a tensor. 4

(d) What are Christoffel's symbols? Write down the transformation laws of Christoffel's symbols of first and second kinds. 4

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