PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STS–101 (C)

(Real Analysis)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that the intersection of three sets is associative. 5

(b) Define supremum and infimum of a set. Find the supremum and infimum of

$$\frac{(-1)^n}{n} : n \quad N \qquad 5$$

(c) Define the set of real numbers R as a complete ordered field. 5

2. *(a)* State and prove the Heine-Borel theorem. 8

(b) Define a closed set with examples. Show that the union of a finite number of closed sets is a closed set. 7

UNIT—II

3. *(a)* Show that every bounded sequence has a limit point. 7

(b) State and prove Cauchy's general principle of convergence for a sequence. 8

4. *(a)* Prove that a monotonic sequence is convergent iff it is bounded. 5

(b) Show that the positive term series $1 \quad r \quad r^2 \quad \cdots$ converges for $r \quad 1$ and diverges to for $r \quad 1$. 5

(c) Test the convergence of the series

$$\frac{(2n)!}{(n!)^2} \; x^n ; \; x \quad 0 \qquad 5$$

UNIT—III

5. *(a)* Let f be a function on R defined by

$$f(x) \quad 1, \quad \text{when} \quad x \quad \text{is rational}$$
$$1, \quad \text{when} \quad x \quad \text{is irrational}$$

Show that f is discontinuous at every point of R. 7

(b) Explain the concept of an uniformly continuous function. Show that the function $f(x) \quad \frac{x}{x \quad 1}$ is uniformly continuous for $x \quad [0, 2]$. 8

6. *(a)* Show that the series

$$\sum_{n \quad 1} \frac{x}{n(1 \quad nx^2)}$$

converges uniformly. 7

(b) Find the Fourier series generated by the periodic function $|x|$ of period 2 . Also compute the values of the series at 0, 2 and 3 . 8

UNIT—IV

7. *(a)* Verify whether the function $f(x) \quad \sin x$ in $[0, \quad]$ satisfies the conditions of Rolle's theorem and hence find c as prescribed by the theorem. 5

(b) State and prove the Lagrange's mean value theorem. Also provide the geometrical interpretation of this theorem. 10

8. *(a)* Using Taylor's theorem, prove that

$$1 \quad x \quad \frac{x^2}{2} \quad e^x \quad 1 \quad x \quad \frac{x^2}{2} e^x; \; x \quad 0$$ 5

(b) Examine the following function for extreme values : 6

$$f(x, y, z) \quad 2x^2 \quad 3y^2 \quad 4z^2 \quad 3xy \quad 8z$$

(c) Obtain the Maclaurin's series expansion of $\sin x$. 4

UNIT—V

9. *(a)* State and prove the Fubini's theorem for double integrals. 10

(b) Evaluate : 5

$$\int_0^1 \int_0^1 \int_0^1 (x \quad y \quad z) \, dx \, dy \, dz$$

10. *(a)* Define improper integrals. Explain the concepts of convergence at the left end and convergence at the right end for improper integrals. 7

(b) Show that

$$\int_0^1 x^{m \quad 1} (1 \quad x)^{n \quad 1} dx$$

exists, if and only if, m and n are both positive. 8

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