PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STSCC–102

(Linear Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Is $\mathbb{R} \setminus \mathbb{Q}$ a field? Justify. Give an example of an infinite dimensional vector space. Express (1, 3) as a linear combination of (1, –1) and (0, 1). 1+1+2=4

(b) Let $S \quad \{(1,2,3), (0,1,2), (0,0,1)\}$. Prove that—

(i) S is linearly independent;

(ii) $L(S) \quad \mathbb{R}^3$. 3+2=5

(c) Prove that intersection of two subspaces of a vector space is a subspace. Give an example to show that union of two subspaces of a vector space need not be a subspace. 4+1=5

2. *(a)* Prove that

$$W \quad \{(a,b,c) \quad \mathbb{R}^3 : a \quad b \quad c \quad 0\}$$

is a subspace of $\mathbb{R}^3(\mathbb{R})$. 4

(b) Find a homogeneous system of linear equations whose solution set W is spanned by the set $S \quad \{(1, \quad 2, 0, 3), (1, \quad 1, \quad 1, 4), (1, 0, \quad 2, 5)\}$. 4

(c) Let S be a linearly independent subset of a vector space $V(F)$. Prove that either S is a basis of V or S can be extended to form a basis of V. 6

UNIT—II

3. *(a)* Prove that

$$S \quad \{(1,2,3), (1 3, \quad 2, \quad 3), (0, 3, \quad 2)\}$$

is an orthogonal basis of $\mathbb{R}^3(\mathbb{R}^3)$. 6

(b) Let $V(F)$ be a vector space and let E be a projection operator on V. Let $U \quad \operatorname{Im} E$ and $W \quad \ker E$. Prove that

(i) if $u \quad U$, then $E(u) \quad U$;

(ii) if $E \quad I$, I being the identity operator on V, then E is singular;

(iii) $V \quad U \quad W$. 2+2+4=8

4. (a) Consider $\mathbb{R}^3(\mathbb{R})$. Let $u \quad (1, \quad 2)$, $v \quad (2, 3)$. Find the projection of u along v. 1

(b) Let V be an inner product space. Prove that—

(i) $||u|| \quad 0$ for all $u \quad V$ and $||u|| \quad 0 \quad u \quad 0$;

(ii) $||ku|| \quad |k|||u||$ for all $u \quad V$, where k is a scalar. 2+2=4

(c) State and prove Cauchy-Schwarz inequality. 1+4=5

(d) Use Gram-Schmidt orthogonalization process to find the orthogonal basis corresponding to the following basis of $\mathbb{R}^3(\mathbb{R})$:

$$S = \{(1,1,1),(0,1,1),(0,0,1)\}$$ 4

UNIT—III

5. (a) Prove that the characteristic roots of a real symmetric matrix are real. 4

(b) Let

$$A \quad \begin{matrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{matrix}$$

Find rank of A. Is A invertible? Justify. 5

(c) Let

$$B \quad \begin{matrix} 4 & 1 & 1 \\ 2 & 5 & 2 \\ 1 & 1 & 2 \end{matrix}$$

Find the characteristic roots of B. 5

6. (a) State and prove Cayley-Hamilton theorem. 1+4=5

(b) Find the minimal polynomial of the following matrix :

$$A \quad \begin{matrix} 2 & 2 & 5 \\ 3 & 7 & 15 \\ 1 & 2 & 4 \end{matrix}$$ 5

(c) Let $v_1, v_2, \ldots, v_n$ be non-zero characteristic vectors corresponding to distinct characteristic values $_1, \ _2, \ldots, \ _n$ of a matrix A. Prove that $v_1, v_2, \ldots, v_n$ are linearly independent. 4

UNIT—IV

7. *(a)* Prove that a generalized inverse of a matrix always exists and is not unique. 4

(b) Prove that a necessary and sufficient condition that the quadratic form $Q \quad X^T AX$ is positive definite is that all characteristic roots of A are positive. 5

(c) Reduce the quadratic form

$$Q \quad X^T AX \quad 4x^2 \quad 9y^2 \quad 2z^2 \quad 6xy \quad 8yz \quad 6zx$$

into cannonical form. 5

8. *(a)* Let A be a matrix such that

$$P \quad XAY$$

where X and Y are two generalized inverses of A. Prove that P is a reflexive generalized inverse of A. 5

(b) Let

$$A \quad \begin{matrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{matrix}$$

Find the Moore and Penrose generalized inverse A of A. 5

(c) Investigate the nature of the quadratic form.

$$Q \quad X^T AX \quad 8x^2 \quad 7y^2 \quad 3z^2 \quad 12xy \quad 8yz \quad 4xz$$

from the characteristic roots of A. 4

UNIT—V

9. *(a)* Suppose that x is a linear combination of $q_1, q_2, \ldots, q_k$ and each of $q_1, q_2, \ldots, q_k$ is itself a linear combination of $r_1, r_2, \ldots, r_l$. Then prove that x is a linear combination of $r_1, r_2, \ldots, r_l$. 6

(b) Solve the following linear equations by using any method : 8

$$\begin{matrix} x_1 & x_2 & 2x_3 & 5 \\ & 2x_1 & x_2 & 2 \\ x_1 & 8x_2 & x_3 & 3 \\ x_1 & 5x_2 & 12x_3 & 4 \end{matrix}$$

10. *(a)* Consider the equations

$$\begin{matrix} 4x_1 & 3x_2 & 2x_3 & 8x_4 & {}_1 \\ 3x_1 & 4x_2 & x_3 & 7x_4 & {}_2 \\ x_1 & x_2 & x_3 & x_4 & {}_3 \end{matrix}$$

Show that the equations admit a solution for any arbitrary ${}_1$, ${}_2$ and ${}_3$. Obtain the total number of linear independent solutions. 7

(b) Solve the following system of equations by using Gauss-Jordan method : 7

$$x \quad 2y \quad z \quad 3$$
$$2x \quad 5y \quad z \quad 4$$
$$3x \quad 2y \quad z \quad 5$$

★ ★ ★