PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STSCC–103

(Probability Theory)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* For a given sequence $\{A_n\}$, prove that

(i) $\limsup A_n \quad \bigcap_{m\ 1} \bigcup_{n\ m} A_n$;

(ii) $\liminf A_n \quad \bigcup_{m\ 1} \bigcap_{n\ m} A_n$. 6

(b) If $\mathcal{H}$ is a field of subsets of , then show that—

(i) $\mathcal{H}$;

(ii) $\mathcal{H}$;

(iii) $E, F \quad \mathcal{H} \quad E \quad F \quad \mathcal{H}$. 1+1+2=4

(c) Given a measure space (, $\mathcal{B}$,); for every sequence $\{E_n\}$ in $\mathcal{B}$. Show that

$$\bigcup_{n\ 1} E_n \qquad (E_n) \qquad n\ 1$$ 4

2. *(a)* Given a measure space (, $\mathcal{B}$,), where is finite, for every sequence $\{E_n\}$ in $\mathcal{B}$. Show that—

(i) lim sup (E_n) $(\limsup E_n)$;

(ii) lim inf (E_n) $(\liminf E_n)$. 5

(b) State and prove Hahn decomposition theorem. 4

(c) Define Lebesgue measure. How can probability be considered as a special type of measure? 5

UNIT—II

3. *(a)* Define measurable function. Given a measure space (, $\mathcal{B}$,). Show that if f is $\mathcal{B}$-measurable if and only if $f^{-1}(A)$ $\mathcal{B}$ for every Borel set A of R. 5

(b) Given $n(\ 1)$ events $A_1, A_2, \ldots, A_n$. Prove that

$$P \bigcup_{i\ 1}^{n} A_i \qquad \sum_{i\ 1}^{n} P(A_i)$$ 5

(c) An urn consists of 6 red and 4 black balls. Two balls are drawn without replacement. What is the probability that the second ball is red if it is known that the first drawn ball is red? 4

4. (a) Define simple function. Show that, if f is a simple function integrable over each of the pairwise disjoint measurable sets $E_1, E_2, ..., E_n$, then f is integrable over

$$E \bigcup_{i=1}^{n} E_i \text{ and } \int_E f = \sum_{i=1}^{n} \int_{E_i} f$$ 5

(b) State and prove Bayes' theorem. 4

(c) State and prove Bonferroni's inequality. 5

UNIT—III

5. (a) What do you mean by probability distribution? Explain discrete and continuous probability distributions. 5

(b) Let X be random variable with p.d.f. $f(x) = C(1 - x^2)$, $0 < x < 1$, then find the following : 4

(i) C (ii) $E(X)$

(iii) Median of X (iv) $P\left(\frac{1}{2} < X < \frac{3}{4}\right)$

(c) A box contains 2^n tickets among which $\binom{n}{r}$ bear the number r, ($r = 0, 1, 2, ..., n$). A group of m tickets is drawn at random from the box, and if the random variable X denotes the sum of the numbers on the tickets drawn, show that

$$E(X) = \frac{mn}{2} \text{ and } \text{var}(X) = \frac{mn}{4}\left(1 - \frac{m-1}{2^n - 1}\right)$$ 5

6. (a) Define expectation for both discrete and continuous cases. 4

(b) The diameter of an electric cable X is a continuous random variable with p.d.f. $f(x) = Kx(1 - x)$, $0 < x < 1$. Find the following : 2+1+2=5

(i) The value of K

(ii) C.d.f. of X

(iii) The value of a such that $P(X < a) = 2P(X > a)$

(c) Let

$$f(x, y) = \begin{cases} 1, & \text{when } 0 < x, y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability that—

(i) $X < \frac{1}{2}, Y > \frac{1}{2}$;

(ii) $X > 2Y$;

(iii) $X = Y$, given $Y = X$. 2+1+2=5

UNIT—IV

7. *(a)* State the properties of moment-generating function. Also prove any two properties of the moment-generating function. 6

(b) Show that

$$M_{XY}(t_1, t_2) \quad M_X(t_1) \quad M_Y(t_2)$$

only if X and Y are independent each other. 3

(c) If (t) is the characteristic function of the variate X, then prove that

(i) $(0) \quad 1$

(ii) $(t) \quad \overline{(t)}$

(iii) $| (t) | \quad 1$

(iv) (t) is uniformly continuous, for all values of t. 5

8. *(a)* State and prove the inversion theorem of the characteristic function. 10

(b) Write a note on Polya-Eggenberger distributions. 4

UNIT—V

9. *(a)* State and prove weak law of large numbers. 2+4=6

(b) State and prove Khintchine's law of large numbers. 2+6=8

10. *(a)* Explain the concept of central-limit theorem. 2

(b) State and prove the generalized weak law of large numbers. 7

(c) Examine if the law of large numbers holds for the sequence of independent random variables $\{X_n\}$ with the distribution of X_n given by

$$f_n(x) \quad \frac{1}{|x|^3}, \quad |x| \quad 1$$
$$0 \quad , \quad \text{otherwise}$$

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