PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STSCC–104

(Distribution Theory)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Consider a sequence of binomial random variables, $X_n \sim$ binomial(n, p), $n = 1, 2, \cdots$. Let one of these random variables be randomly selected. $Y = X_N$ is a new random variable, where N is an integer-valued random variable which does not depend on X's. Find the distribution of Y. 7

(b) If $X_1 \sim$ Poisson$(\ _1)$ and $X_2 \sim$ Poisson$(\ _2)$, prove that the conditional distribution of X_1, given that $X_1 + X_2 = n$, is

$$\text{binomial}\ \left(n, \frac{\ _1}{\ _1 + \ _2}\right)\ \text{distribution}$$

Also, prove the inverse of this problem. 7

2. *(a)* Define hypergeometric distribution. Obtain the mean and variance of the distribution. Derive binomial distribution as a limiting case of hypergeometric distribution. 1+2+4=7

(b) Let X_1 and X_2 be independent random variables following geometric distribution with parameters p_1 and p_2. Find the distribution of $Y = X_1 - X_2$. 7

UNIT—II

3. *(a)* Define normal distribution. With the help of m.g.f., show that normal distribution is a limiting form of binomial distribution. 2+5=7

(b) If X and Y are independent random variables from $N(0, \ ^2)$, show that X / Y follows Cauchy distribution. 7

4. *(a)* Show that the exponential variate inherits the memoryless property from the geometric probability law. 7

(b) Let

$X_1 \sim$ Gamma (a, b_1) and $X_2 \sim$ Gamma (a, b_2),

then show that $X_1 \quad X_2$, X_1 / X_2 and $X_1 / (X_1 \quad X_2)$ are independent variate. 7

UNIT—III

5. *(a)* If the bivariate random variables $X \quad (X_1, X_2)$ follow bivariate normal distribution with mean vector 0 and dispersion matrix

$$\begin{matrix} 1 & \\ & 1 \end{matrix}$$

then find the probability density function of X_1 / X_2. 7

(b) If χ^2_1 and χ^2_2 are two independent χ^2-variate with n_1 and n_2 degrees of freedom respectively, then χ^2_1 / χ^2_2 is a $_2$-variate with $\frac{n_1}{2}, \frac{n_2}{2}$ variate. 7

6. *(a)* Find the relation between F and chi-square distribution. 8

(b) Show that the transformation

$$\frac{\frac{1}{2} F}{1 \quad \frac{1}{2} F}$$

changes the F-distribution to beta-distribution. 6

UNIT—IV

7. *(a)* Derive non-central χ^2 as a mixture of central χ^2 with $(2m \quad n)$ d.f. 8

(b) State and prove additive or reproductive property of non-central χ^2. 6

8. *(a)* Define compound Poisson distribution. Obtain the probability mass function of left and right truncated Poisson distribution. 8

(b) Write a note on compound distribution. Obtain m.g.f. of compound distribution. 2+4=6

UNIT—V

9. *(a)* Define order statistics. Prove that the density function of the sum of all the order statistics is simply the density function of the underlying distribution multiplied by the sample size. 2+5=7

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample from a population with continuous density. Show that

$$Y \quad \min(X_1, X_2, \cdots, X_n)$$

is exponential with parameter n iff each X_i is exponential with parameter . 7

10. *(a)* Find the density function of the r-th order statistics. 3

(b) Write a note on mixture distribution. 5

(c) Define the three types of extreme value distribution. 6

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