PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(3rd Semester)

Course No. : STSCC–302

(Statistical Inference—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define the likelihood ratio test. Derive the two-tailed likelihood ratio test for mean of a normal population when variance is unknown. 1+6=7

(b) Deduce the one-tailed ratio test for the mean of a normal population. 7

2. *(a)* Deduce the likelihood ratio test for the equality of means of two normal populations. 9

(b) Define similar critical region and similar test. 5

UNIT—II

3. *(a)* Explain how the posterior proportional to the likelihood multiplied by the prior is obtained from the Bayes theorem. 6

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(, r)$, where r is the precision, but the value of r is unknown. Find the posterior distribution of r given the sample. 5

(c) Explain the concept of estimation, testing of hypothesis and decision making using the concept of action space. 3

4. *(a)* Define the absolute error loss function. Obtain the Bayes estimate of a under the absolute error loss function. 5

(b) If $X_1, X_2, \ldots, X_n$ follows Bernoulli distribution with parameter and prior distribution $\frac{1}{2}, \frac{1}{2}$, then calculate the estimate of , which minimizes the squared error loss function. 5

(c) Let X follows Bin(n, p) and let $L(p, d(x))$ $[p \quad d(x)]^2$. Let (p) 1 for $0<p<1$ be the prior p.d.f. of p. Obtain the Bayes estimate of p and then calculate the corresponding Bayes risk. 4

UNIT—III

5. *(a)* Let $X_1, X_2, \ldots, X_n$ be i.i.d random sample drawn from a normal $N(, 1)$; Q R and let their prior p.d.f. $( )$ of is $N(0, 1)$. Find $(1 \quad)$ level Bayes confidence interval for . 5

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \ ^2)$. Then derive the minimal information prior. 6

(c) Define the following : 3

(i) Uniform prior

(ii) Improper prior

6. *(a)* Deduce the Bayes factor for testing a simple null hypothesis against a simple alternative hypothesis. 4

(b) Let $\underset{\frown}{X}$ $(X_1, X_2, \ldots, X_n)$ be a random sample from $N(, \ ^2)$; ^{2} is known. If the prior distribution of is $g( )$ 1, then the posterior distribution of given $\underset{\frown}{X}$ $\underset{\frown}{x}$ is $N\ \bar{x}, \frac{^2}{n}$. Prove it. 5

(c) If $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \ ^2)$, then derive the minimal information prior. 5

UNIT—IV

7. *(a)* Explain the sequential procedure of testing of hypothesis. What are its advantages over the traditional method of testing of hypothesis? 3+3=6

(b) If for the choice A $\frac{1}{\quad}$ and B $\frac{\quad}{1}$ the SPRT terminates with probability 1 is of strength $(\ , \ )$, then prove that

$$\frac{\quad}{1}; \qquad \frac{\quad}{1}$$

and

$$(\qquad) \qquad .$$ 8

8. *(a)* Prove that the SPRT terminates with probability 1 both under H_0 and H_1. 7

(b) State and prove the fundamental identity of sequential analysis. 7

UNIT—V

9. *(a)* Explain the Wilcoxon matched pair signed-rank test. 7

(b) Twelve students appeared for two papers in Physics. The marks obtained by them are given below :

Student	*Paper–I*	*Paper–II*
A	30	70
B	30	70
C	25	68
D	27	63
E	23	52
F	21	50
G	27	68
H	23	59
I	23	52
J	30	70
K	28	70
L	25	64

Use Kendall's rank correlation coefficient to test the independence between the marks. 7

10. *(a)* Explain the purpose, assumptions and working of the two-sample Kolomogorov-Smirnov test. 7

(b) Explain how run-test can be used to measure the randomness of a data set. 7

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