PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(3rd Semester)

Course No. : STSCC–303

(Stochastic Process)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define Markov process. 2

(b) Classify the states of a Markov chain. 7

(c) Consider a Markov chain with state space S (0, 1, 2, 3). Classify the chain

$$P \begin{matrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \quad 5$$

2. *(a)* Prove that if state k is persistent null then for every chain $p_{jk}^{(n)}$ 0 as n .

If state k is aperiodic non-null, then

$$p_{jk}^{(n)} \quad \frac{F_{jk}}{kk} \text{ as } n \quad 5$$

(b) Define communication state of a Markov chain. Write the properties of communication. 1+2=3

(c) Discuss higher order transition probability with respect to a Markov chain. 6

UNIT—II

3. *(a)* What is a Poisson process? Write down the postulates of a Poisson process. 2+3=5

(b) Show that the rate of occurrence of more than one event in a very small interval is zero. 9

4. *(a)* State and prove the continuation property of a Poisson process. 7

(b) Write notes on the following : 3½×2=7

(i) Birth-death process

(ii) Martingale

UNIT—III

5. *(a)* Define one-dimensional random walk and absorbing barriers of a random walk. 5+2=7

(b) Find the probability distribution of a random walk which is moving a distance to the right if head occurs and a distance to the left if tail occurs in a coin tossing experiment. 7

6. *(a)* In Gambler's ruin problem, find the expected number of bets that a gambler has to make starting at i, $0 \quad i \quad N$, N is the total sum involved in the game. 12

(b) Define correlated random walk. 2

UNIT—IV

7. *(a)* Discuss Brownian process. 7

(b) Show that in a branching process, the probability function is given by $P_n(S) \quad P_{n\ 1}[P(S)]$ and $P_n(S) \quad P[P_{n\ 1}(S)]$. 7

8. *(a)* In a branching process, we have

$$P[X_n \quad k] \quad bC^{k\ 1};\ k \quad 1, 2, \ldots$$

Find the distribution of the number of offsprings of the nth generation. 8

(b) What is branching process? Give examples. 2+2=4

(c) Define chance of extinction. 2

UNIT—V

9. *(a)* What is a renewal process? Give example. 3+1=4

(b) Define the term 'stopping time'. 3

(c) Show that the renewal function M satisfies the equation

$$M(t) \quad F(t) \quad \int_0^t M(t \quad x)\, dF(x)$$ 7

10. *(a)* State and prove Wald's equation. 5

(b) Show that the distribution of $N(t)$ is given by

$$P_n(t) \quad P[N(t) \quad n] \quad F_n(t) \quad F_{n\ 1}(t)$$

and the expected number of renewals by

$$M(t) \quad \sum_{n\ 1} F_n(t)$$ 5

(c) Write a note on equilibrium renewal process. 4

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