PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STS–102 (C)

(Linear Algebra)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* What is a field? Explain with examples. 2+3=5

(b) Prove that in any field F

$$x_1 \quad x_2 \quad \cdots \quad x_n \qquad \sum_{i=1}^{r} x_i \qquad \sum_{i=r+1}^{n} x_i$$

for any r with $1 \quad r \quad n$ and also show that the sum in the left side can be reordered in any manner. 6

(c) If F is a field, then prove that $a_0 \quad 0$ for all $a \quad F$, and if neither a nor b is zero, then $ab \quad 0$. 4

2. *(a)* Define a vector and a vector space. Show that in a vector space, there can only be one zero vector. Also prove that the additive inverse of a vector is always unique. 2+3+3=8

(b) Describe linearly independent and linearly dependent vectors. 3

(c) Define the basis and dimension of a vector space. 4

UNIT—II

3. *(a)* State and prove Gram-Schmidt orthogonalization process. 6

(b) Define unit vector and orthogonal vector. Describe the process of normalization of a vector. Is

$$X \quad \begin{matrix} 2 \\ 4 \\ 4 \end{matrix}$$

a unit vector? 1+1+2+2=6

(c) Define orthogonal transformation of vectors. 3

4. *(a)* Prove that the distance between two vectors remains invariant under orthogonal transformation. 5

(b) Describe the application of projection in least squares theory. 6

(c) Construct an orthonormal basis from the given basis

$$x_1 \begin{matrix} 2 \\ 3 \\ 0 \end{matrix}, \; x_2 \begin{matrix} 6 \\ 1 \\ 0 \end{matrix} \text{ and } x_3 \begin{matrix} 0 \\ 2 \\ 4 \end{matrix}$$ 4

UNIT—III

5. *(a)* Define idempotent matrix. Let $A_{n\ n}$ and $B_{n\ n}$ be two matrices, then show that AB A and BA B, A and B are idempotent matrices. 1+4=5

(b) Prove that, if $_1, _2, \cdots, _n$ are the characteristic roots of A, then A KI will have the characteristic roots as $_1$ K, $_2$ $K, \cdots, _n$ K, where K is a scalar. 5

(c) Prove that the characteristic roots of an orthogonal matrix are either +1 or –1, provided they are real. 5

6. *(a)* Define the geometric and algebraic multiplicities of a matrix A with characteristic root . Also show that the geometric multiplicity of a characteristic root cannot exceed its algebraic multiplicity. 2+3=5

(b) Define minimal polynomial. If A and B be n n matrices, and B is non-singular, then show that A and $B^{\ 1}AB$ have the same minimal polynomial. 1+4=5

(c) State and prove the Cayley-Hamilton theorem. 5

UNIT—IV

7. *(a)* Define the quadratic form of a matrix. Show that every quadratic form over a field F in n variables $x_1, x_2, \cdots, x_n$ can be expressed in the form $X\,BX$, where X $[x_1, x_2, \cdots, x_n]$ is a column vector and B is a symmetric matrix of order n over the field F. 1+4=5

(b) Explain the signature and index of a real quadratic form of a matrix. State and prove the Sylvester's law of inertia. 2+4=6

(c) Define generalized inverse of a matrix. Show that a generalized inverse always exists and is not unique. 1+3=4

8. (a) Explain different classes of generalized inverse. Show that, if A^r $A^{g_1}AA^{g_2}$, where A^{g_1} and A^{g_2} are two g-inverses, then A^r is a reflexive generalized inverse. 2+4=6

(b) Define definite, semidefinite and indefinite real quadratic forms. 4

(c) Determine a non-singular matrix P such that PAP is a diagonal matrix, where

$$A \quad \begin{matrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{matrix}$$

Interpret the result in terms of quadratic forms. 5

UNIT—V

9. (a) Does the following system of equations possess a common non-zero solution? 6

$$\begin{matrix} x & 2y & 3z & 0 \\ 3x & 4y & 4z & 0 \\ 4x & 10y & 12z & 0 \end{matrix}$$

(b) Prove that if the number of equations (linear and homogeneous) is less than the number of unknowns, then the equation must have a non-trivial solution. 6

(c) Define homogeneous and nonhomogeneous linear equations. 3

10. (a) Consider a system of nonhomogeneous linear equations $A_{m\ m}X_{n\ 1}$ $B_{m\ 1}$ and (A) r; r $\min(m, n)$. Then state and prove the necessary and sufficient condition for the system of equations to be consistent. 8

(b) Consider the equations

$$\begin{matrix} 4x_1 & 3x_2 & 2x_3 & 8x_4 & & 1 \\ 3x_1 & 4x_2 & x_3 & 7x_4 & & 2 \\ x_1 & x_2 & x_3 & x_4 & & 3 \end{matrix}$$

Show that the equations admit a solution for any arbitrary $_1$, $_2$, $_3$. Obtain the total number of linear independent solutions. 7

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