PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STS–104 (C)

(Methods in Mathematical Statistics—I)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Show that Bernoulli distribution is negatively skewed when $p \ \frac{1}{2}$ and positively skewed when $p \ \frac{1}{2}$. Also show that $_1 \ \ 0$, when $p \ \frac{1}{2}$. 7

(b) From the m.g.f. of Binomial distribution, find the mean and variance of the distribution. 4

(c) Show that the conditional distribution of X_1 given $X_1 \ \ X_2 \ \ n$; $(X_1, X_2) \sim$ Poisson $(\ _1, \ _2)$ is distributed as Binomial variate. 4

2. *(a)* Explain how you will use hyper-geometric model to estimate the number of fish in a lake. 5

(b) Define geometric distribution. Find the mean and variance of the distribution with the help of m.g.f. 2+3=5

(c) Find the m.g.f. of multinomial distribution. 5

UNIT—II

3. *(a)* Prove that the linear combination of independent normal variate is also a normal variate. 5

(b) Define exponential distribution. Find the mean and variance of the distribution. Show that the sum of independent exponential variate is a gamma variate. 2+2+6=10

4. *(a)* Random variables X and Y have joint p.d.f. as

$$f(x, y) \quad 24xy; \quad x, y \quad 0, x \quad y \quad 1$$
$$\quad\quad 0; \quad\quad\quad otherwise$$

Show that $U \quad X \quad Y$ and $V \quad X/Y$ are independent $_1$ and $_2$ variates respectively. 5

(b) Show that exponential distribution lacks memory. 5

(c) Show that a r.v. X is Cauchy distribution with parameter $(\; , \;)$ iff X^{1} follows Cauchy distribution with parameters

$$\frac{}{(\;^2 \quad ^2)}, \frac{}{(\;^2 \quad ^2)}$$ 5

UNIT—III

5. *(a)* Prove that (X, Y) possesses a bivariate normal distribution iff every linear combination of X and Y, viz., $aX \quad bY$, $a \quad 0$, $b \quad 0$ is a normal variate. 6½

(b) Prove that for a bivariate normal distribution, the marginal distribution is equal to univariate distribution. 8½

6. *(a)* Prove that the regression equations of Y on X and X on Y are linear and homoscedastic. 7

(b) State and derive bivariate normal distribution. 8

UNIT—IV

7. *(a)* Obtain the characteristic function of non-central ^{2}-distribution. 7

(b) Give the complete derivation of non-central t-distribution. 8

8. *(a)* Define compound Poisson distribution. Obtain the probability mass function of left and right truncated Poisson distributions. 3+6=9

(b) Write a note on truncation of density functions. 3

(c) Obtain the p.m.f. of left truncated Binomial distribution at the point a. 3

UNIT—V

9. *(a)* Define order statistics. Show that if $X_1, X_2, \cdots, X_n$ be random sample of size n from $p(x)$ and $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ are the order statistics, then $U(r) \quad P[X_{(n)}]$, $r \quad 1, 2, \cdots, n$ is the rth order statistics in a sample of size n from uniform distribution. 2+5=7 *(b)* A random sample of size 4 is drawn from the discrete uniform distribution $P(X \quad x) \quad 1/6, \; x \quad 1, 2, \cdots, 6$. Obtain the distribution function of the smallest and largest order statistics. 3

(c) Find the density function of the sample range for a sample of size 2 from $N(\; , \quad ^2)$ and hence find the expected value of the range. 5

10. *(a)* Obtain the distribution of the sample range R in a sample of size n from the rectangular population

$$dF \quad \frac{dx}{\quad}, 0 \quad x$$

Find $E(R)$ and $V(R)$. 4+2+2=8

(b) Show that the sampling distribution of the area under a probability density function between two-order statistics is independent of the form of the density function. 7

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