PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(3rd Semester)

Course No. : STS–302 (C)

(Statistical Inference—II)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Explain the likelihood ratio test principle. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\ ,\ ^2)$ population. We wish to test $H_0 :\ ^2\ \ ^2_0$ (specified) to be tested against $H_1 :\ ^2\ \ ^2_0$. Deduce the corresponding likelihood ratio test. 1+6=7

(b) Find an -level likelihood ratio test of $H_0 : p\ \ p_0$ against $H_1 : p\ \ p_0$ based on a random sample of size 1 from binomial (n, p). 8

2. *(a)* Deduce the likelihood ratio test for the equality of variance of two normal populations. 8

(b) Let X have a p.m.f.

$$P_M(X\ \ x)\ \ \frac{\binom{M}{x}\binom{N\ \ M}{n\ \ x}}{\binom{N}{n}},\ x\ \ 0, 1, \cdots, M$$

Find the UMP test of size for testing $H_0 : M\ \ M_0$ against $H_1 : M\ \ M_0$ (M_0 is a fixed, known integer). 7

UNIT—II

3. *(a)* Define the posterior distribution. Suppose the random variable X takes two values 0 and 1 having the p.m.f.

$$f(x|\ )\ \ 1\ \ \text{if}\ \ x\ \ 0$$
$$\text{if}\ \ x\ \ 1$$

Also the parameter is a discrete random variable having p.m.f.

$$g(\ )\ \ 0\ 2\ \text{if}\ \ 0\ 3\ \text{or}\ \ 0\ 7$$
$$0\ 6\ \text{if}\ \ 0\ 5$$

Then find the posterior distribution for . 2+5=7

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample $N(, 1)$ and $( ) \sim N(0, 1)$. Find the posterior distribution of $(\underset{\sim}{x})$. 4

(c) Define *(i)* loss function and *(ii)* Bayes risk. 4

4. *(a)* Define the LINEX loss function. Obtain the Bayes estimate of from the LINEX loss function. 1+4=5

(b) Let $X_1, X_2, \cdots, X_n$ be n observations from $B(1,)$ and $( ) \sim$ beta $(,)$. Also we have, the loss function

$$L(, a) \quad \frac{(\quad a)^2}{(1 \quad)}$$

Compute the value of which minimizes the loss function. 5

(c) Let $(X_1, X_2, \cdots, X_n)$ be iid observations from

$$f(x|) \quad \frac{^x e}{(x)!}; \; x \quad 0, 1, 2, \cdots$$

Obtain the Bayes risk for estimating under the squared error loss function. 5

UNIT—III

5. *(a)* Define the Bayesian confidence interval. Let $X_1, X_2, \cdots, X_n$ be iid Bin(1,) random variable and let the prior distribution of on is $u(0, 1)$. Find (1) level Bayes confidence interval for . 1+5=6

(b) Define *(i)* Jeffreys' invariant prior and *(ii)* Minimal information prior. 4

(c) There are two boxes B_1 and B_2. B_1 has four slips and B_2 has twenty slips, and the slips are labelled 1, 2, 3, 4 and 1, 2, ⋯, 20 respectively. The boxes are equally likely and so are the slips inside it. Find out the Bayes factor in favour of choosing a slip of number 2 from B_1. 5

6. *(a)* Obtain the Bayes factor for testing a composite null versus a composite alternative hypothesis. 6

(b) Let $X_1, X_2, \cdots, X_n$ be iid random sample drawn from a normal $N(, 1)$; $Q \quad R$ and let their prior p.d.f. $( )$ of is $N(0, 1)$. Find (1) level Bayes confidence interval for . 5

(c) Explain the concept of Bayesian prediction. 4

UNIT—IV

7. *(a)* Describe the process of testing of hypothesis using the sequential procedure. How is it better than the traditional method of testing of hypothesis? 3+2=5

(b) Show that if the SPRT of strength (,) and boundary points (A, B) terminates with probability 1, then

$$A \quad \frac{1}{\quad}, \; B \quad \frac{\quad}{1}$$

$$\text{or } A \sim \frac{1}{\quad} \text{ and } B \sim \frac{\quad}{1} \qquad 6$$

(c) Explain the efficiency of SPRT. 4

8. *(a)* Show that for any random variable n which takes the values 0, 1, 2, ⋯

$$E(n) \qquad P(n \quad m) \qquad 4$$

$$n \quad 1$$

(b) Let $E(|Z||H_0)$ and $E(|Z||H_1)$. Obtain the appropriate expression for ASN under H_0, H_1. Show that the expressions are

$$E(n|H_0) \quad \frac{b(1 \quad) \quad a}{E(Z|H_0)}$$

$$E(n|H_1) \quad \frac{b \quad (1 \quad)}{E(Z|H_1)}$$

where (,) is the strength of SPRT. 6

(c) Give an example to explain the economy of sequential testing. 5

UNIT—V

9. *(a)* What are the disadvantages of non-parametric tests? 5

(b) Explain the Kolmogorov-Smirnov one-sample test for goodness of fit. 6

(c) Explain the Mann-Whitney U test. 4

10. *(a)* Explain the Kendall's rank correlation test. 5

(b) Explain the Wald-Wolfowitz run test. 5

(c) Define a kernel. Explain its role in non-parametric estimation. 5

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