PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(3rd Semester)

Course No. : STS–303 (C)

(Stochastic Process)

Full Marks : 75

Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define stochastic process with examples. 2+2=4

(b) Consider a Markӧv chain consisting of four states with t.p.m. (transition probability matrix)

$$P \quad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 2 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 3 & 0 & 0 & 0 & 1 \end{array}$$

Verify whether the chain is reducible. 6

(c) Define the following : 1½+1½+2=5

(i) Irreducible chain

(ii) Absorbing state

(iii) Communicative state

2. *(a)* Show that in a Markӧv chain, trials are made dependent for condition probability and correlation coefficient between states. 8

(b) Prove that—

(i) if state j is persistent, non-null, then as n $p_{jj}^{(nt)}$ $\frac{t}{}$, where state j is period t and as n ;

(ii) $p_{jj}^{(nt)}$ $\frac{1}{}$, where state j is aperiodic.

3½+3½=7

UNIT—II

3. *(a)* What is a counting process? Write down the properties of a counting process. When is a counting process said to be independent? 2+2+2=6

(b) Write down the properties of a Poisson process. 2

(c) Show that in an interval of length zero, probability of occurrence of k events is zero. 7

4. *(a)* Define supermartingales and submartingales. 2+2=4

(b) State and prove the decomposition property of a Poisson process. 8

(c) Write a note on birth-death process. 3

UNIT—III

5. *(a)* Write a note on one-dimensional random walk. 5

(b) Explain the gambler's ruin problem. 10

6. *(a)* Write a note on three-dimensional random walk. 3

(b) Show that

$$\text{corr}\,[X_n, X_{n\ \ 1}]$$
$$\frac{(1 \ \ 2a) \ \ 2(a \ \ b)_{pn \ \ 1} \ \ (2_{pn \ \ 1} \ \ 1)(2_{pn} \ \ 1)}{4(p_n q_n \ \ p_{n \ \ 1} q_{n \ \ 1})}$$

where X_n denotes the direction of movement to left or right corresponding to the value of –1 or +1 at the nth step. The t.p.m. is given by

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{X_n} \\ & & 1 & 1 \\ X_{n \ \ 1} & 1 & 1 \ \ a & a \\ & 1 & b & 1 \ \ b \end{array}$$

$$0 \quad a, b \quad 1$$

–1 and +1 are the values assumed by the r.v. X_n; $n \quad 0, 1, 2, \cdots$. 12

UNIT—IV

7. *(a)* Discuss branching process. 7

(b) In a branching process, find the distribution of the number of offsprings of the nth generation, where

$$P[X_n \quad k] \quad bC^{k \ \ 1}; \ k \quad 1, 2, \cdots$$ 8

8. *(a)* If $m \quad E(X_1)$ and $\ ^2 \quad \text{var}(X_1)$, then show that $E(X_n) \quad m_n$, and

$$\text{var}(X_n) \quad \begin{array}{ll} \dfrac{m^{n \ \ 1}(m^n \ \ 1)}{(m \ \ 1)} \ ^2, & \text{if} \ \ m \quad 1 \\ n \ ^2, & \text{if} \ \ m \quad 1 \end{array}$$ 9

(b) Write a note on Brownian process. 6

UNIT—V

9. *(a)* Write a note on renewal process. 5

(b) Show that with probability 1,

$$\frac{N(t)}{t} \quad \frac{1}{\ } \text{ as } t$$

where $\quad E(X_n) \quad .$ 7

(c) Define stopping time. 3

10. *(a)* State and prove the elementary renewal theorem. 8

(b) Write a note on stationary renewal process. 7

★ ★ ★