PG Odd Semester (CBCS) Exam., December—2016

STATISTICS

(1st Semester)

Course No. : STSCC–101

(Real Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Give an example of a set which is neither bounded above nor bounded below. Prove that the set of natural numbers $\mathbb{N}$ is not bounded above. 1+4=5

(b) Show that union of two open sets in $\mathbb{R}$ is an open set. 4

(c) State Bolzano-Weierstrass theorem on infinite sets. Show that O is a limit point of $\frac{1}{n} : n \quad \mathbb{N}$. 1+4=5

2. *(a)* Let A, B, C be sets. Prove that

$A \setminus (B \quad C) \quad (A \setminus B) \quad (A \setminus C)$. 4

(b) Let $A \quad (0,1)$. Show that $\inf A \quad 0$. 3

(c) State and prove Heine-Borel theorem. 1+6=7

UNIT—II

3. *(a)* Prove that limit of a sequence, if it exists, is unique. 4

(b) State Sandwich theorem on convergent sequences. Use it to prove—

(i) $\frac{(\ 1)^n}{n}$ converges to 0;

(ii) $\frac{\sin n}{n}$ converges to 0. 1+2+2=5

(c) Test the convergence of the following series : 5

$$\frac{x}{1\ 2}\ \frac{x^2}{2\ 3}\ \frac{x^3}{3\ 4}\ \frac{x^4}{4\ 5}\ \cdots;\ x>0.$$

4. (a) Prove that every convergent sequence in $\mathbb{R}$ is bounded. Is the converse true? Justify your answer. 4+1=5

(b) Use Cauchy's root test to test the convergence of the following series : 3

$$\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{16}\ \cdots$$

(c) Prove that every Cauchy sequence in $\mathbb{R}$ converges. 6

UNIT—III

5. (a) Prove that the function $f:\mathbb{R}\ \ \mathbb{R}$ defined by

$$f(x)\ \begin{array}{ll} 1, & \text{if } x\ \ \mathbb{Q} \\ 0, & \text{if } x\ \ \mathbb{R}\backslash\mathbb{Q} \end{array}$$

is not continuous at any point of $\mathbb{R}$. 5

(b) Define uniform continuity of a function. Give an example of a continuous function which is not uniformly continuous. Prove that any function $f:\mathbb{N}\ \ \mathbb{R}$ is uniformly continuous. 1+1+3=5

(c) State Cauchy-Hadamard theorem. Determine the radius of convergence of the power series

$$\begin{array}{l} n^3 x^n \\ n\ \ 1 \end{array}$$ 1+3=4

6. (a) Prove that a function differentiable at a point of its domain, is continuous there. Is the converse true? Justify your answer. 3+1=4

(b) Find the Fourier series expansion of the function f with period $2\ $, defined as

$$f(x)\ \begin{array}{ll} 1, & \text{if}\ \ \ x\ \ 0 \\ 1, & \text{if}\ 0\ \ x \end{array}$$

Also calculate the sum of the series at $x\ \ \frac{}{2}$. 4+1=5

(c) Let $A\ \ \mathbb{R}$, $f, g:A\ \ \mathbb{R}$ be two functions. Let f and g be both continuous at a point $c\ \ A$. Prove that both $f\ \ g$ and fg are continuous at c. 5

UNIT—IV

7. *(a)* State Rolle's theorem. Verify Rolle's theorem for the function $f:[\ 2,2]\quad \mathbb{R}$ defined by $f(x)\quad x^3\quad 4x\quad x\quad [\ 2,2]$. 1+4=5

(b) Use Taylor's theorem to prove that

$$\sin x\quad x\quad \frac{x^3}{3!}\quad \frac{x^5}{5!}\quad \cdots$$

5

(c) Show that $f:\mathbb{R}^2\quad \mathbb{R}$ defined by

$f(x,y)\quad x^3\quad y^3\quad 12xy\quad 63(x\quad y)\quad (x,y)\quad \mathbb{R}^2$

has a maxima at (–7, –7) and a minima at (3, 3). 4

8. *(a)* State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f:[0,1]\quad \mathbb{R}$ defined by

$f(x)\quad x^2\quad 2x\quad 3\quad x\quad [0,1]$ 1+3=4

(b) Show that $f:\mathbb{R}\quad \mathbb{R}$ defined by

$f(x)\quad x^5\quad 5x^4\quad 5x^3\quad 1\quad x\quad \mathbb{R}$

has a maxima at $x=1$, a minima at $x\quad 3$ and neither a maxima nor a minima at $x=0$. 5

(c) Find the minimum value of $x^2\quad y^2\quad z^2$ subject to $ax\quad by\quad cz\quad p$. 5

UNIT—V

9. *(a)* Prove that

$$(m,n)\quad \frac{(m)\quad (n)}{(m\quad n)}$$

4

(b) Evaluate

$$\int_c (x^2\quad y^2)dxdy$$

where c is the circle $x^2\quad y^2\quad a^2$. 4

(c) State Fubini's theorem. Prove that

$$\int_0 \frac{\tan^{\ 1}(ax)}{1\quad x^2}dx\quad \frac{}{2}\log(1\quad a);a\quad 0$$

1+5=6

10. *(a)* Prove that

$$\frac{1}{2}\quad \sqrt{\ }$$

5

(b) Show that

$$\int_0^1\int_0^1 \frac{x\quad y}{(x\quad y)^3}dxdy\quad \int_0^1\int_0^1 \frac{x\quad y}{(x\quad y)^3}dydx$$

4

(c) State Dirichlet's theorem. Prove that

$$\frac{dxdydz}{\sqrt{1 \quad (x^2 \quad y^2 \quad z^2)}} \quad \frac{2}{8}$$

where integration is extended over all positive values of the variables for which the expression is real. 1+4=5

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