PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(1st Semester)

Course No. : MTM–101 (C)

(Algebra—I)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that $a \sim b$ iff a b $\mathbb{Q}$ is an equivalence relation on $\mathbb{R}$. 5

(b) Let G be a group such that $O(a)$ 2 for all a G. Show that G is Abelian. 5

(c) Show that every subgroup of a cyclic group is cyclic. 5

2. *(a)* Let $f : \mathbb{R}$ $\mathbb{R}$ and let A $\mathbb{R}$. Show that $f^{-1}(f(A))$ A iff f is one-one. 5

(b) Show that intersection of any class of subgroups of a group is a subgroup of the group. 5

(c) Show that every cyclic group is Abelian. Is $(\mathbb{Q},)$ cyclic? Justify. 2+3=5

UNIT—II

3. *(a)* Find the order and parity of the permutation

(1 2) (1 4 7) (3 5) (3 6) (8 9 10 11) 5

(b) If

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 3 & 1 & 6 & 2 \end{matrix}$$

then find ^{1001}. 5

(c) Show that A_4 has no subgroup of order 6. 5

4. *(a)* Find the number of elements of order 4 in S_6. 5

(b) Show that the centre of the group S_3 is $\{e\}$, where e is the identity in S_3. 5

(c) Let H be a subgroup of G such that the index of H in G is 2. If a, b $G \setminus H$, then show that ab H. 5

UNIT—III

5. *(a)* Show that $SL(2, \mathbb{R})$ is a normal subgroup of $GL(2, \mathbb{R})$, where $GL(2, \mathbb{R})$ is the group of all 2 2 invertible real matrices and $SL(2, \mathbb{R})$ is the group of all 2 2 real matrices with unit determinant. 5

(b) Let G be a group and let H, K be normal subgroups of G. Show that

$$HK / H \quad K / H \quad K$$ 5

(c) Show that A_n is a simple group for all n 5. 5

6. *(a)* Let H be a subgroup G. Show that the normalizer $N(H)$ of H is a subgroup of G and H is a normal subgroup of $N(H)$. 5

(b) Find the number of homomorphisms from S_3 to $\mathbb{Z}_6$. 5

(c) Find all the non-isomorphic groups of order 6. 5

UNIT—IV

7. *(a)* Let R be a ring. Show that the following conditions are equivalent : 5

(i) R has no non-zero nilpotent elements

(ii) For all $a \quad R$, if $a^2 \quad 0$, then $a \quad 0$

(b) Show that every finite commutative ring without zero divisors is a field. 5

(c) Let R and R be rings and $f : R \quad R$ be on to ring homomorphism. Show that

$$R \quad R / \ker f$$

where $\ker f \quad \{x \quad R : f(x) \quad O$ the zero element of $R\}$. 5

8. *(a)* Show that a ring R with 1 is a Boolean ring iff $(a \quad b)ab \quad 0$, where Boolean ring means a ring each element of which satisfies $x^2 \quad x$. 6

(b) Show that the field of quotients of $\mathbb{Z}$ is $\mathbb{Q}$. 6

(c) Show that there does not exist any ring homomorphism $f : \mathbb{R} \quad \mathbb{R}$ such that, $\ker f \quad \mathbb{Q}$. 3

UNIT—V

9. *(a)* Let $c[0, 1]$ denote the ring of all continuous function $f : [0, 1] \quad \mathbb{R}$ with respect to addition and multiplication defined by

$$(f \quad g)(x) \quad f(x) \quad g(x) \quad f, g \quad c[0, 1]$$

and $x \quad [0, 1]$

$$(fg)(x) \quad f(x)g(x) \quad f, g \quad c[0, 1]$$

and $x \quad [0, 1]$

Prove or disprove that

$$\{f \quad c[0, 1] : f(\tfrac{1}{2}) \quad 0\}$$

is a maximal ideal of $c[0, 1]$. 6

(b) Show that every ideal of a Euclidean domain is principal. 5

(c) Show that in an integral domain every prime element is irreducible. 4

10. *(a)* Show that $\mathbb{Z}$ is an ED. 5

(b) In $\mathbb{Z}[i]$, find g.c.d. of $9 \quad 5i$ and $\quad 9 \quad 13i$. 6

(c) Prove or disprove : In a PID R, if p and q are two prime elements such that p / q, then p and q are associates. 4

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