PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(1st Semester)

Course No. : MTMCC–104

(Numerical Analysis)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Derive Stirling formula using central difference operator. 7

(b) Calculate the nth divided difference of $\frac{1}{x}$ based on the points $x_0, x_1, x_2, \cdots, x_n$. 7

2. *(a)* Let x $(0\ a_1\ a_2 \cdots a_n\ a_{n\ 1}\cdots)$ ^{c}, where a_1 0 and 0 a; 1 for all i 1; and e is an integer, $fl(x)$ $(0\ a_1 a_2 \cdots a_n)$ ^{e} the n-digit normalised floating point approximation to x obtained by rounding, then prove that

$$\frac{|x \quad fl(x)|}{|x|} \quad \frac{1}{2} \quad {}^{n\ 1}$$

7

(b) Prove the following : 4+3=7

(i) $^{3}y_{1/2} \quad y_2 \quad 3y_1 \quad 3y_0 \quad y_{\ 1}$

(ii) ^{2}

UNIT—II

3. *(a)* Derive Simpson's $\frac{1}{3}$rd rule. 7

(b) Show that Simpson's $\frac{1}{3}$rd rule integrates all polynomials of degree 3 exactly. 7

4. *(a)* Derive 2-point Gauss-Legendre rule. 7

(b) Compute the integral

$$\int_0^1 e^{\ x^2}\, dx$$

by trapezoidal rule, Simpson's $\frac{1}{3}$rd rule and Simpson's $\frac{3}{8}$th rule and compare the computed results with the exact value of the integral. 7

UNIT—III

5. *(a)* Derive secant method and find the order of convergence. 7

(b) Show that the number N of iterations required to guarantee an approximation to a root to an accuracy by the bisection method is given by

$$N \quad 2\frac{\log(/ l)}{\log 2}$$

where l is the length of the initial inverval containing the root of the equation. 7

6. *(a)* Discuss Jacobi method and prove the iteration scheme convergence if the coefficient matrix of the system of linear equations is strictly diagonally dominant. 7

(b) Solve the system of linear equations

$$\begin{array}{rrrr} & 2x_1 & x_2 & 1 \\ x_1 & 2x_2 & x_3 & 1 \\ & x_2 & 2x_3 & 1 \end{array}$$

by Cholesky's method. 7

UNIT—IV

7. *(a)* Solve by Picard's method

$$\frac{dy}{dx} \quad \frac{x^2}{y^2 \quad 1}; \; y(0) \quad 0$$

Find out y for x 0 25 and 0 5 correct to three decimal places. 7

(b) Derive Runge-Kutta method of order 2. 7

8. *(a)* Find the exact solution of

$$\frac{dy}{dx} \quad 2 \quad \frac{y}{x}, \; y(1) \quad 2$$

Use Euler's method to determine $y(1\ 2)$ with h 0 1 and h 0 05. Compare the numerical results with the exact solution. 7

(b) Derive the Taylor series method of order n. 7

UNIT—V

9. *(a)* Given $f(x) \quad c\ 3^x \quad x\ 3^{x\ 1}$. Find the corresponding difference equation. 6

(b) If E is the operator and k is any constant, then prove the following : 4+4=8

(i) $(E \quad k) f(x) \quad k^{x\ 1} \quad \frac{f(x)}{k^x}$

(ii) $(E \quad k)^n f(x) \quad k^{x\ n\ n} \frac{f(x)}{k^x}$

10. *(a)* If $f_1(x), f_2(x), \cdots, f_n(x)$ are the n solutions of the nth order homogeneous difference equation with constant coefficients i.e.,

$a_0 f(x \quad n) \quad a_1 f(x \quad n \quad 1) \cdots a_n f(x) \quad 0$

then prove that

$$_1 f_1(x) \quad _2 f_2(x) \cdots \quad _n f_n(x)$$

is also a solution of this equation, where $_1, \ _2, \cdots, \ _n$ are some arbitrary constants. 7

(b) Solve $2y_{k \ 2} \quad 5y_{k \ 1} \quad 2y_k \quad 0$. Also find the particular solution when $y_0 \quad 0$ and $y_1 \quad 1$. 7

★ ★ ★