PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(1st Semester)

Course No. : MTMCC–105

(Operations Research and Optimization Techniques—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Using revised simplex method, solve the following IPP : 7

Max $Z = 2x_1 + x_2$

subject to

$$3x_1 + 4x_2 \leq 6$$
$$6x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

(b) Discuss the effects of any changes in the requirement vector b on the optimality and feasibility of the solutions of the following LPP : 7

Max $Z = Cx$

subject to

$$Ax \leq b,\ x \geq 0,\ x \in \mathbb{R}^n,\ A \in \mathbb{R}^{m \times n}$$

2. *(a)* The processing times in hours for each job are given in the following table. Determine the sequence for the jobs that will minimize the total elapsed time. Also find the idle time for each machine. It is given that jobs must be processed on the three machines A, B and C in the order ABC : 7

Machine	Job	1	2	3	4	5
	A	5	7	6	9	5
	B	2	1	4	5	3
	C	3	7	5	6	7

(b) Obtain the variation in C_1 and C_2 which are permitted without changing the optimal solution of the following LPP : 7

Max $Z = x_1 + 2x_2$

subject to

$$x_1 + x_2 \leq 3$$
$$x_1 + 2x_2 \leq 5$$
$$3x_1 + x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

UNIT—II

3. *(a)* If C_g (P, S, A) be a two-player zero-sum game, then prove that a strategy pair (e_i, e_j) is an equilibrium strategy, iff

$$e_i^T A e_j \quad \max_{k \ \{1, 2, ..., m\}} \min_{l \ \{1, 2, ..., n\}} A_{kl} \quad \min_{l \ \{1, 2, ..., n\}} \max_{k \ \{1, 2, ..., m\}} A_{kl}$$

where, $A \quad \mathbb{R}^{m \ n}$. 8

(b) Using dominance property, solve the game whose payoff matrix is given below : 6

		Player B				
		B_1	B_2	B_3	B_4	B_5
Player A	A_1	2	5	10	7	2
	A_2	3	3	6	6	4
	A_3	4	4	8	12	1

4. *(a)* Solve the game with the following payoff matrix by graphical solution method : 7

		Player B	
		B_1	B_2
Player A	A_1	1	2
	A_2	5	4
	A_3	7	9
	A_4	4	3
	A_5	2	1

(b) Solve the game with the following payoff matrix by reducing it to LPP : 7

		Player B	
		B_1	B_2
Player A	A_1	1	1
	A_2	1	1

UNIT—III

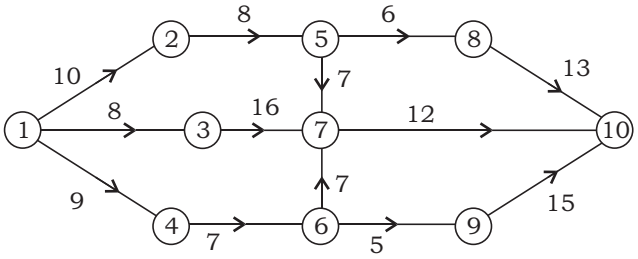
5. *(a)* Discuss the replacement model whose maintenance cost increases with time and the value of the money remains the same during the entire period. 8

(b) Let the value of money be assumed to be 10% and suppose that the machine M_1 is replaced after every 3 years, whereas machine M_2 is replaced after every 6 years. The yearly costs of both the machines are given below :

Year	1	2	3	4	5	6
Machine M_1	1000	200	400	1000	200	400
Machine M_2	1700	100	200	300	400	500

Determine which machine should be purchased. 6

6. *(a)* Identify the critical path in respect to the following network : 7



Activity times in number of days are indicated on the network.

(b) A project composed of 7 activities whose time estimates are listed below :

Activities	*Time in weeks*		
	t_o	t_m	t_p
1–2	1	1	7
1–3	1	4	7
1–4	2	2	8
2–5	1	1	1
3–5	2	5	14
4–6	2	5	8
5–6	3	6	15

Draw the network, calculate the expected variance for each activity and find the expected project completion time. 4+2+1=7

UNIT—IV

7. *(a)* Derive the EOQ model with uniform rate of demand, infinite production rate and having no shortage. 8

(b) An item is produced at the rate of 60 items per day. The demand occurs at the rate of 35 units per day. If the setup cost is ₹ 120 per run and holding cost is ₹ 0·02 per unit of item per day, find the economic lot size for one run, assuming that the shortages are not permitted. 6

8. *(a)* Derive the EOQ model with uniform rate of demand, finite rate of production and having shortages which are to be fulfilled. 9

(b) The demand for an item and the time period of consumption is given below. The carrying cost is ₹ 2 per unit and the ordering cost is ₹ 75 per order. Calculate the economic order quantity and the minimum cost of inventory. 5

Demand of units	35	50	45	30	80
Period in months	1	2	3	2	6

UNIT—V

9. *(a)* Derive Gomory's cut for all integer programming problem : 7

Max Z Cx

subject to

Ax b where A $\mathbb{R}^{m\ n}$, x $\mathbb{Z}^n$

x 0

(b) Solve the following problem by Gomory's cutting plane method : 7

Max Z $7x_1$ $9x_2$

subject to

x_1 $3x_2$ 6

$7x_1$ x_2 35

x_1, x_2 0

x_2 is an integer.

10. *(a)* Solve the following all integer programming problem by Gomory's method : 6

Max Z x_1 $4x_2$

subject to

$2x_1$ $4x_2$ 7

$5x_1$ $3x_2$ 15

x_1, x_2 0

and are integers.

(b) Use branch- and-bound method to solve the following problem : 8

Max Z $8x_1$ $5x_2$

subject to

x_1 x_2 6

$9x_1$ $5x_2$ 45

x_1, x_2 0

x_1, x_2 are integers.

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