PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(3rd Semester)

Course No. : MTMCC–304

(Operations Research and Optimization Techniques—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define Binomial distribution and Poisson distribution. If on an average 8 ships out of 10 arrive safely at a port, find the mean and standard deviation of the number of ships arriving safely out of a total of 1600 ships. 4+3=7

(b) A bakery keeps stock of popular brand of cake. Daily demand based on past experience is given below :

Daily demand	0	15	25	35	45	50
Probability	0·01	0·15	0·20	0·50	0·12	0·02

Consider the following sequence of random numbers :

48, 78, 09, 51, 56, 77, 15, 14, 68, 09

Using the sequence simulate the demand for the next 10 days. 7

2. *(a)* The automobile company manufactures around 50 scooters. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions :

Production (per day)	146	147	148	149	150	151	152	153	154
Probability	0·04	0·09	0·12	0·14	0·11	0·10	0·20	0·12	0·08

The finished scooters are transported in a specially designed lorry accommodating 150 scooters. Using random numbers

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57

simulate the process to find out the following :

(i) What will be the average number of scooters waiting in the factory?

(ii) What will be the average number of empty space on the lorry? 3+4=7

(b) *(i)* During the course of a day, a machine turns out either 0, 1 or 2 defective pens with probability $\frac{1}{6}$, $\frac{2}{3}$ and $\frac{1}{6}$ respectively. Calculate the mean value and the variance of the defective pens produced by the machine in a day. 4

(ii) Write the properties of normal distribution. 3

UNIT—II

3. *(a)* Show that, if the arrival pattern of the queuing model follows Poisson distribution, then the inter-arrival time follows the exponential distribution. 7

(b) Derive and solve the steady state equations of model $(M/E_k/1):(1/FCFS)$. 7

4. *(a)* Derive and solve the difference equations for queuing model $(M/M/1):(\quad/FCFS)$. 9

(b) Customers arrive at a sales counter manned by a single person according to a Poisson process with mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer. 5

UNIT—III

5. *(a)* Derive the condition for optimum stock level for the single period model with instantaneous demand and time independent costs. 7

(b) *(i)* What are the different criteria in decision theory under the conditions of uncertainty? 2

(ii) A newspaper boy has the following probabilities of selling a magazine :

No. of copies sold	10	11	12	13	14
Probability	0·10	0·15	0·20	0·25	0·30

Cost of a copy is 30 paise and sale price is 50 paise. He cannot return unsold copies. How many copies should he order? Obtain the result by using EMV criterion. 5

6. *(a)* Derive the condition for optimum stock level for the single period without setup cost and uniform demand in discrete case. 8

(b) A manager has a choice between *(i)* a risky contract promising ₹ 7 lakh with probability 0·6 and ₹ 4 lakh with probability 0·4, and *(ii)* a diversified portfolio consisting of two contracts with independent outcomes each promising ₹ 3·5 lakh with probability 0·6 and ₹ 2 lakh with probability 0·4. Construct a decision tree by using EMV criteria. Can you arrive at the decision using EMV criteria? 6

UNIT—IV

7. (a) Define goal programming problem. An office equipment manufacturer produces two kinds of products—chairs and lamps. Production of either a chair or a lamp requires 1 hour of production capacity in the plant. The plant has a maximum capacity of 10 hours per week. The gross margin from the sale of a chair is ₹ 80 and ₹ 40 for that of a lamp. Formulate the problem as a GPP if the goal of the firm is equally ranked as follows : 1+6=7

(i) To earn a profit of ₹ 800 per weeks.

(ii) Because of the limited sales capacity, the maximum number of chairs and lamps that can be sold are 6 and 8 per week respectively.

(b) A firm produces two products A and B. Each product must be processed through two departments namely 1 and 2. Department 1 has 30 hours of production capacity per day, and department 2 has 60 hours. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2. The management has set the goals in determining the daily product mix based on priority level as follows :

P_1 : Minimize the under achievement of joint total production of 10 units

P_2 : Minimize the under achievement of producing 7 units of product B

P_3 : Minimize the under achievement of producing 8 units of product A

Formulate this problem as a GP model and then solve it by using the graphical method. 7

8. Use modified simplex method to solve the following GP problem : 14

$$\text{Minimize } Z \quad P_1 d_1 \quad P_2(2d_2 \quad d_3) \quad P_3 d_1$$

subject to the constraints

$$x_1 \quad x_2 \quad d_1 \quad d_1 \quad 400$$

$$x_1 \quad d_2 \quad 240$$

$$x_2 \quad d_3 \quad 300$$

$$x_1,\ x_2,\ d_1,\ d_1,\ d_2,\ d_3 \quad 0.$$

UNIT—V

9. *(a)* Solve the following nonlinear programming problem using Lagrangian multipliers : 7

$$\text{Optimize } Z \quad 4x_1 \quad 9x_2 \quad x_1^2 \quad x_2^2$$

subject to

$$4x_1 \quad 3x_2 \quad 15$$
$$3x_1 \quad 5x_2 \quad 14$$
$$x_1, x_2 \quad 0$$

(b) Use the Kuhn-Tucker conditions to solve the following nonlinear programming problem : 7

$$\text{Maximize } Z \quad 7x_1^2 \quad 6x_1 \quad 5x_2^2$$

subject to

$$x_1 \quad 2x_2 \quad 10$$
$$x_1 \quad 3x_2 \quad 9$$
$$x_1, x_2 \quad 0$$

10. *(a)* What are the different methods for solving nonlinear programming problem? 4

(b) Use Wolf's method to solve the following quadratic programming problem : 10

$$\text{Maximize } Z \quad 2x_1 \quad x_2 \quad x_1^2$$

subject to

$$2x_1 \quad 3x_2 \quad 6$$
$$2x_1 \quad x_2 \quad 4$$
$$x_1, x_2 \quad 0$$

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