2016/ODD/08/23/MTM–305/479

PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(3rd Semester)

Course No. : MTMCC–305

(Classical Mechanics and Electrodynamics)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define a scleronomic system. Establish that, for a scleronomic dynamical system of n degrees of freedom, the kinetic energy represents a homogeneous quadratic function of generalized velocities. 1+3=4

(b) Deduce the Lagrange's equations of motion of a dynamical system of n degrees of freedom specified by n generalized coordinates $q_j (j \quad 1, 2, \cdots, n)$ and moving under a conservative field of force. 7

(c) A particle of mass m moves on a plane in the field of force given by

$$\vec{F} \qquad kr\cos \quad \hat{r}$$

where k is a constant. Find the equation of motion of the particle. 3

2. *(a)* What is action principle? Derive Euler-Lagrange equations of motion from a variational principle. 1+6=7

(b) Use Hamilton's principle to obtain the equations of motion of a particle of mass m in a spherical coordinate system. 7

UNIT—II

3. *(a)* Why is it necessary to adopt the Hamilton-Jacobi equation to solve a dynamical system? Use Hamilton-Jacobi theory to develop the Hamilton-Jacobi equation. 2+6=8

(b) The Hamiltonian of a linear harmonic oscillator is given by

$$H(q,\ p) \quad \frac{1}{2m}(p^2 \quad m^2 \quad {}^2q^2)$$

where is the frequency of oscillation. Show that the function given by

$$S(q,\ \ ,\ t) \quad \frac{m}{2}(q^2 \qquad {}^2)\cot \quad t \quad m \quad q \quad \operatorname{cosec} \quad t$$

is a solution of the HJ equation. Establish that, this function generates a correct solution to the motion of the harmonic oscillator. 6

4. (a) How Jacobi's theorem justifies the principal function as a complete integral of the H-J equation? 4

(b) Solve the motion of 1-D simple harmonic oscillator. 6

(c) A particle of mass m moves in 1-D subject to the potential described as

$$V \qquad \frac{a}{\sin^2 \dfrac{x}{x_0}}$$

Construct the H-J equation and hence obtain an integral form for the principal function. 4

UNIT—III

5. (a) Show that, the angular momentum of a moving body about a point is the algebraic sum of the angular momentum of the centre of mass about the point and the angular momentum of the whole body about the centre of mass. What will happen if the point coincides with the centre of mass? 4+1=5

(b) What is force-free motion of a rigid body? Establish that, in case of a force-free motion of a rigid body, the Euler's dynamical equations produce two constants of motions. 1+2=3

(c) A body rotates under no forces about its one point, the principal moments at this point are 4 , 3 and , where 0. Initially, the angular velocity of the body has components $_1$ n, $_2$ 0 and $_3$ $n\sqrt{2}$ about the principal axes. Find the angular velocity components at any time t. 6

6. (a) Obtain a total time derivative of a vector with respect to a set of rotating frame of references. Hence, interpret the Coriolis theorem using the previous relation. 3+3=6 *(b)* Establish Euler's dynamical equations of motion for the torque-free motion of a rigid body. 8

UNIT—IV

7. *(a)* What is a Hamiltonian function? Give physical interpretation of Hamiltonian function and establish it analytically. 2+3=5

(b) Show that the set of transformations

$$Q \quad \ln \frac{1}{q} \sin p$$

$p \quad q \cot p$ is canonical, and hence obtain the corresponding generating function. 5

(c) A particle of mass m is subjected to a potential $V(x) \quad \frac{k}{x}, k \quad 0$. If H is a Hamiltonian and L is a Lagrangian of the system, calculate $[H, V(x)]$ and $[L, H]$. 2+2=4

8. *(a)* Obtain the Hamiltonian of the motion of a simple pendulum and hence determine its equations. 4+1=5

(b) Establish that, Poisson's bracket is invariant under canonical transformations. 5

(c) State and prove Jacobi-Poisson theorem. 4

UNIT—V

9. *(a)* Define the following terms : 1+2+1+1=5

(i) Surface and volume density of charges

(ii) Electrostatic field and electric intensity vector

(iii) Current density

(iv) Electric displacement vector

(b) Explain why, the classical theory is not adequate to handle the theory of electromagnetism. 4

(c) What are gauge transformations? Show that, the scalar potential function and vector potential function A are gauge invariants. 5

10. *(a)* Discuss the transformation properties of electric and magnetic field vectors. 5

(b) Establish that, Maxwell's electromagnetic field equations are invariant under Lorentz transformations. 9

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