PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(1st Semester)

Course No. : MTM–103 (C)

(Ordinary Differential Equations)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Examine the existence and uniqueness of solution of the following IVP, where $f(x, y) \quad x^2 |y|, \quad |x| \quad 1, |y| \quad 1$: 5

$$\frac{dy}{dx} \quad f(x, y), \quad y(0) \quad 0$$

(b) *(i)* Reduce the following differential equation by an equivalent system of first-order equations : 2

$$y \quad (x) \quad y \quad (x) \quad x^2 (y \quad (x))^2$$

(ii) Find the general solution of the system

$$\frac{dx}{dt} \quad x \text{ and } \frac{dy}{dt} \quad y$$ 3

(c) Determine the critical point(s) and discuss the nature and stability properties of the critical point(s) for the system

$$\frac{dx}{dt} \quad 4x \quad 2y \text{ and } \frac{dy}{dt} \quad 5x \quad 2y$$ 1+2+2=5

2. *(a)* Reduce the equation

$$x^2 p^2 \quad py(2x \quad y) \quad y^2 \quad 0$$

where p stands for $\frac{dy}{dx}$ to Clairaut's form and find its complete primitive and also its singular solution. 6

(b) Solve the following system of equations : 5

$$\frac{dx}{dt} \quad 3x \quad 4y$$

$$\frac{dy}{dt} \quad x \quad y$$

(c) A system of non-linear differential equations is given by

$$\frac{dx}{dt} \quad y(x^2 \quad 1) \text{ and } \frac{dy}{dt} \quad 2xy^2$$

Obtain the critical point(s), differential equations of the paths and hence solve this equation to find the paths. 1+1+2=4

UNIT—II

3. (a) Define eigenvalues and eigenfunctions relating to a BVP. Find the eigenvalues and the eigenfunctions for the equation $y \quad y \quad 0$ subject to the conditions $y(0) \quad 0 \quad y(2\ )$. 2+4=6

(b) Solve the equation

$$y \quad 2x \quad \frac{3}{x} \ y \quad 4y \quad 0$$

by finding a simple solution of the adjoint equation by inspection. 5

(c) Construct Green's function for the BVP

$$x\ (t) \quad e^t;\ x(0) \quad 0 \quad x(1)$$ 4

4. (a) What is a Strum-Liouville problem? Establish that any two eigenfunctions of a Strum-Liouville problem with respect to a weight function is always orthogonal. Can it be orthonormal always? Justify your cliam. 2+4+2=8

(b) Using an appropriate Green's function, solve the BVP

$$x\ (t) \quad 1 \quad t;\ x(0) \quad 0;\ x\ (1) \quad 1$$ 7

UNIT—III

5. (a) Find the indicial equation and its roots for the following differential equation : 3+1=4

$$4x^2y\ (x) \quad (2x^4 \quad 5x)y\ (x) \quad (3x^2 \quad 2)y(x) \quad 0$$

(b) Determine the nature of the point x for

$$(1 \quad x^2)y\ (x) \quad 2xy\ (x) \quad p(p \quad 1)y \quad 0$$

where $p \quad \mathbb{R}$. 4

(c) Show that $x(t) \quad J_0(2it)$ is the solution of the differential equation

$$t^2x\ (t) \quad tx\ (t) \quad 4t^2x \quad 0$$ 7

6. (a) Find the solution on $(\ 1, 1)$ for the IVP

$$(1 \quad t^2)\,x\ (t) \quad 2tx\ (t) \quad 12x(t) \quad 0$$

subject to $x(0) \quad 1,\ x\ (0) \quad 0.$ 6

(b) Show that the hypergeometric equation

$$(t \quad t^2)x\ (t) \quad [c \quad (a \quad b \quad 1)] \quad x\ (t) \quad abx(t) \quad 0$$

where a, b, c are constants has 0, 1 and as regular singular points and hence find its solution(s) in terms of hypergeometric functions. 3+6=9

UNIT—IV

7. *(a)* If $F(t)$ has period $T \quad 0$, then show that

$$L\{F(t)\} \quad \frac{1}{1 \quad e^{pT}} \int_0^T e^{\ pT} F(t)\, dt$$

4

(b) Find $L\{J_0(t)\}$, where $J_0(t)$ is the Bessel's function of order zero. 4

(c) Use Laplace transform method to solve the BVP

$$y\ (t) \quad 9y(t) \quad 18t$$

if $y(0) \quad 0 \quad y\ \frac{}{2}$. 7

8. *(a)* Find the inverse Laplace transform of

$$f(p) \quad \frac{p^2}{(p^2 \quad 4)^2}$$

4

(b) Solve the following by using Laplace transform method : 6

$$y\ (t) \quad ty\ (t) \quad y(t) \quad 0;\ y(0) \quad 0,\ y\ (0) \quad 1$$

(c) Show that

$$\int_0 \frac{\cos\ x}{2\ \ 1}\, dx \quad \frac{}{2} e^{\ x}, \quad x \quad 0$$

5

UNIT—V

9. *(a)* Use shooting method to solve the BVP

$$y\ (0x) \quad y \quad 1$$

subject to $y(0) \quad 0 \quad y(1)$. 7

(b) Develop an appropriate finite difference scheme and solve the equation

$$y\ (x) \quad xy\ (x) \quad 0$$

with the conditions $y(0) \quad 1$, $y\ (0) \quad 0$. Also, compute the value of $y(0\ \ 2)$. 7+1=8

10. *(a)* The data points are given below :

i	:	0	1	2
x_i	:	4	9	16
f_i	:	2	3	4

Estimate the function value f at $x \quad 7$ using cubic splines. 7

(b) Use a finite difference approximation to solve the BVP

$$xy \quad y \quad 0;\ y(1) \quad 1,\ y(2) \quad 2$$

8

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