PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(1st Semester)

Course No. : MTM–104 (C)

(Numerical Analysis)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Derive Newton's divided interpolation formula. 8

(b) Tabulate $f(x)$ 5^x from x $0(1)4$, and show that interpolation fails for x $1/2$. 7

2. *(a)* Let x $(0\ a_1a_2\cdots a_n a_{n\ 1}\cdots)$ ^{e}; where a_1 0 and 0 a_i 1 for all i 1; and e is an integer, $fl(x)$ $(0\ a_1a_2\cdots a_n)$ ^{e}; the n-digit normalized floating point approximation to x obtained by rounding, then prove that

$$\frac{|x \quad fl(x)|}{|x|} \quad \frac{1}{2} \quad {}^{n\ 1}$$

7

(b) Find the interpolating polynomial for the following data : 8

$f(\ 1)$ 0, $f(0)$ 1, $f(1)$ 2

UNIT—II

3. *(a)* Derive Simpson's $\frac{1}{3}$rd rule. 7

(b) Show that Simpson's $\frac{1}{3}$rd rule integrates all polynomials of degree 3 exactly. 8

4. *(a)* Derive two-point Gauss-Legendre rule. 7

(b) Prove that trapezoidal rule is the average of left rectangular rule and right rectangular rule. 8

UNIT—III

5. *(a)* Derive Newton-Raphson method and find its order of convergence. 8

(b) Find an approximation to $\sqrt{3}$ correct to two decimal places using the bisection method. 7

6. *(a)* Derive Jacobi iterative method and condition of convergence. 7

(b) Solve the following system of linear equations by Doolittle method : 8

$$10x_1 \quad x_2 \quad x_3 \quad 12$$
$$x_1 \quad 10x_2 \quad x_3 \quad 12$$
$$x_1 \quad x_2 \quad 10x_3 \quad 12$$

UNIT—IV

7. *(a)* Derive Runge-Kutta method of order two. 7

(b) Solve

$$\frac{dy}{dx} \quad x \quad y; \; y(0) \quad 1$$

by Picard's method. 8

8. *(a)* Derive Euler's iterative forms. 7

(b) Solve

$$\frac{dy}{dx} \quad x \quad y; \; y(0) \quad 1$$

by Euler's method with $h \quad 0\ 1$. 8

UNIT—V

9. *(a)* Given

$$U_x \quad c_1 2^x \quad c_2 3^x \quad \frac{1}{2}$$

Find the corresponding difference equations. 8

(b) If $f_1(x), f_2(x), \cdots, f_n(x)$ are the n solutions of the nth order homogeneous difference equation with constant coefficients i.e.,

$$a_0 f(x \quad n) \quad a_1 f(x \quad n \quad 1) \quad \cdots \quad a_n f(x) \quad 0$$

then prove that

$$_1 f_1(x) \quad _2 f_2(x) \quad \cdots \quad _n f_n(x)$$

is also a solution of this equation, where $_1, \ _2, \cdots \ _n$ are some arbitrary constants. 7

10. *(a)* Solve

$$y_{k\ 4} \quad 4y_{k\ 3} \quad 6y_{k\ 2} \quad 4y_{k\ 1} \quad y_k \quad 0$$ 8

(b) If E is the operator and k is any constant, then prove the following : 7

(i) $(E \quad k) f(x) \quad k^{x\ 1} \quad \frac{f(x)}{k^x}$

(ii) $(E \quad k)^n f(x) \quad k^{x\ n\ n} \quad \frac{f(x)}{k^x}$

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