2016/ODD/08/23/MTM–302 (C)/486

PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(3rd Semester)

Course No. : MTM–302 (C)

(Fluid Mechanics)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Distinguish between Lagrangian and Eulerian methods of describing fluid motion. 3

(b) What is the equation of continuity? Determine the equation of continuity by vector approach for incompressible fluids. 1+6=7

(c) Show that

$$(x^2/a^2) f_1(t) \quad (y^2/b^2) f_2(t) \quad 1$$

where $f_1(t) f_2(t) \quad 1$ is a possible form of the boundary surface of a liquid. 5

2. *(a)* Define streamline and path line of a flow. Obtain the differential equations for streamline and path line of a flow. 2+3=5

(b) What are vorticity and circulation? Find the vorticity components in Cartesian form and hence express the components of vorticity, if z-axis is the axis of rotation. 2+2+1=5

(c) Show that the following velocity profile represents a possible liquid motion :

$$u \quad \frac{2xyz}{(x^2 \quad y^2)^2}, \; v \quad \frac{(x^2 \quad y^2)z}{(x^2 \quad y^2)^2}, \; w \quad \frac{y}{x^2 \quad y^2}$$

Is this motion irrotational? Justify your claim. 5

UNIT—II

3. *(a)* What is stream function? Show that the difference of the values of stream function at two points represents the flux of the fluid across any curve joining the two points. 1+4=5 *(b)* Define sources and sinks of a flow domain. If there are sources at (a, 0) and (a, 0) and sinks at (0, a) and (0, a) all of equal strength, show that the circle through these four points is a streamline. 2+4=6

(c) Find the image of a line source in a circular cylinder. 4

4. *(a)* Define complex potential of a flow domain. A liquid domain is bounded by the axes of x and y in the first quadrant, where a source of strength m at a distance r from the origin on the bisector of the angle xoy is placed. Prove that the complex potential for the region is $m\log(z^4 \quad r^4)$. 2+4=6

(b) Show that the image system of a source outside a circle consists of an equal source at the inverse point and an equal sink at the centre of the circle. 6

(c) Determine the image of a line doublet parallel to the axis of a right-circular cylinder. 3

UNIT—III

5. *(a)* Obtain the velocity potential and stream function, if a sphere is moving in a liquid at rest at infinity. Hence, obtain the equations to the lines of flow. 6+2=8

(b) Show that when a sphere of radius a moves with uniform velocity U through a perfect incompressible infinite fluid, the acceleration of a particle of the fluid at (r, 0) is

$$3U^2 \quad \frac{a^3}{r^4} \quad \frac{a^6}{r^2}$$ 7

6. *(a)* If liquid streaming past a fixed sphere with velocity U, find the velocity potential of the flow and hence determine the lines of flow relative to the sphere. 6

(b) A sphere of radius a is moving with constant velocity U through an infinite liquid at rest at infinity. If p_0 be the pressure at infinity, show that the pressure at any point of the surface of the sphere, the radius to which point makes an angle with the direction of motion is given by

$$p \quad p_0 \quad \frac{1}{2} \quad U^2 \quad 1 \quad \frac{9}{4}\sin^2$$ 9

UNIT—IV

7. *(a)* Define viscous fluid with an example. Starting from the Navier-Stokes equations for the motion of an incompressible fluid moving under conservative forces, prove that the vorticity $\vec{}$ satisfies the differential equation

$$\frac{D\vec{}}{Dt} \quad (\vec{}\;\vec{})\vec{q} \qquad \vec{}^{\,2}$$

being coefficient of kinematic viscosity. 1+7=8

(b) Show that the velocity field

$$u(y) \quad \frac{h^2}{8}\,\frac{dp}{dx} \quad 1 \quad 4\,\frac{y}{h}^{\;2}, \; v \quad 0 \quad w$$

satisfies the equation of motion for the two-dimensional steady flow of a viscous incompressible fluid with constant viscosity and constant pressure gradient. 7

8. *(a)* Find an expression for the rate of dissipation of energy of a liquid due to viscosity. 6

(b) Calculate the velocity distribution in the steady flow of a viscous incompressible fluid along an infinitely long circular pipe due to an applied pressure gradient. Also determine the expressions of volumetric rate of flow and coefficient of friction at the wall of the pipe. 9

UNIT—V

9. *(a)* Discuss Prandtl's theory of the boundary layer and obtain the differential equation of motion in the layer along a plate. 2+5=7

(b) Derive Blasius equations for laminar boundary layer flow of an incompressible viscous fluid along a flat plate. 8

10. *(a)* The velocity distribution in the boundary layer over a flat plate is given by $u \quad U(1 \quad e^{\;n})$, $\quad y/\;.$ Obtain displacement thickness and momentum thickness. 3+3=6

(b) Derive the solution of Von Karman method to obtain the momentum integral equation of the boundary layer. 9

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