2016/ODD/08/23/MTM–304 (C)/488

PG Odd Semester (CBCS) Exam., December—2016

MATHEMATICS

(3rd Semester)

Course No. : MTM–304 (C)

(Classical Mechanics and Tensors)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Show that, in case of a scleronomic, holonomic and conservative systems, the total energy of the systems under Lagrangian mechanics is always conserved. 5

(b) For a 1-D linear harmonic oscillator, obtain the Lagrangian of the system and hence find the equation of motion of the system. 3+2=5

(c) What do you mean by the conservation theorem? Establish that, for conservation of linear momentum, the co-ordinates have to be cyclic. 1+4=5

2. *(a)* Using the method of calculus of variations, obtain the path in brachistochrone problem. 7

(b) Show that the Euler-Lagrange equations of motion are invariant under generalized co-ordinate transformations. 4

(c) Using Hamilton's principle, obtain the equation of motion of a single particle system in plane polar co-ordinate system. 4

UNIT—II

3. *(a)* Obtain the 2-D equations of motion of a rigid body. 8

(b) A body moves under no forces about a point O, the principal moments of inertia at O being $3A, 5A$ and $6A$ respectively. Initially, the angular velocity of the body has components

$_1$ n, $_2$ 0 and $_3$ n about the principal axes. Show that at any later time

$$_2 \quad \frac{3n}{\sqrt{5}} \tanh \frac{nt}{\sqrt{5}}$$

and ultimately the body rotates about the mean axis. 7

4. *(a)* A sphere of radius r rolls down an inclined plane, rough enough to prevent any sliding. Discuss the motions of the sphere. 8

(b) What do you mean by torque-free motion? Discuss the general method of solving Euler's dynamical equations of motion under torque-free condition. 1+6=7

UNIT—III

5. *(a)* Using Hamilton's action principle, derive Hamilton's canonical equations of motions. 6

(b) Prove that the set of transformation equations

$$q \quad e^{Q} \operatorname{cosec} p \text{ and } p \quad \cot^{\;\;1} \frac{p}{q}$$

are canonical. 5

(c) State and prove Poisson's first theorem. 4

6. *(a)* Establish that if the Lagrangian of a holonomic system is not unique so also the corresponding Hamiltonian. 5

(b) Obtain the Hamiltonian of the motion of a planet under an inverse square law of attraction. Also determine its equations of motion. 6

(c) A particle of mass m is acted upon by a constant force F. Show that the co-ordinate x and momentum p are in the forms

$$x \quad x_0 \quad \frac{p_0}{m} t \quad \frac{1}{2} \frac{F}{m} t^2 \text{ and } p \quad p_0 \quad Et$$

where at t 0, x x_0 and p p_0. 4

UNIT—IV

7. *(a)* Define Kronecker delta. Show that Kronecker delta is a mixed type of tensor of rank 2. 1+3=4

(b) Show that $\sqrt{g}\, dx^1 dx^2 \cdots dx^n$ is an invariant. 5

(c) Transform the line element

$$ds^2 \quad dx^2 \quad dy^2 \quad dz^2$$

in cylindrical co-ordinates. 6

8. *(a)* Prove that if all the components of a tensor vanish at a point in one co-ordinate system, they vanish at the same point in all co-ordinate systems. 3

(b) What are Christoffel's symbols of first and second kinds? Show that

$$\frac{g_{ij}}{x^k} \qquad jk,\ i \qquad ik,\ j \qquad 0$$

2+3=5

(c) Establish that the Christoffel's symbol of first kind is not a tensor quantity. Under what condition, it may represent a tensor quantity? 6+1=7

UNIT—V

9. *(a)* Define covariant differentiations of vectors. Show that the covariant derivatives of the fundamental tensors vanish. 2+6=8

(b) A necessary and sufficient condition that the first covariant derivative of a covariant vector be symmetrical is that the vector be gradient. 7

10. *(a)* Define Riemann-Christoffel curvature tensor and obtain an expression for it. 5

(b) A Riemannian V_2 has the metric

$$(ds)^2 \qquad \frac{(dr)^2 \quad r^2(d\ \ )^2}{r^2 \quad a^2} \qquad \frac{r^2(dr)^2}{(r^2 \quad a^2)^2}, \quad r \quad a$$

Prove that the differential equation of the geodesic may be expressed as

$$a\frac{dr}{d}^{\ 2} \qquad a^2r^2 \quad k^2r^4$$

7

(c) Show that the divergence of

$$G \qquad \frac{1}{2}g \ \ G$$

is identically zero. 3

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