UG Even Semester (CBCS) Exam., May—2017

COMPUTER SCIENCE

(2nd Semester)

Course No. : MCS–202 (C)

(Statistical Methods and Applications)

Full Marks : 75
Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* If the letters of the word 'REGULATIONS' are arranged at random, what is the chance that there will be exactly 4 letters between *R* and *E*? 5

(b) State and prove, Boole's inequality theorem. 2+8=10

2. *(a)* The odds that, person *X* speaks the truth are 3 : 2 and the odds that, person *Y* speaks the truth are 5 : 3. In what percentage of cases are they likely to contradict each other on an identical point? 6

(b) For any three events *A*, *B* and *C*, prove that

(i) $P(A \quad B|C) \quad P(A|C) \quad P(B|C) \quad P(A \quad B|C)$

(ii) $P(A|C) \quad P(A \quad \overline{B}|C) \quad P(A \quad B|C)$ 5

(c) Let *A* and *B* be two events, such that $P(A) \quad 3/4$ and $P(B) \quad 5/8$. Show that

(i) $P(A \quad B) \quad 3/4$

(ii) $3/8 \quad P(A \quad B) \quad 5/8$ 4

UNIT—II

3. *(a)* A problem in statistics is given to the three students *A*, *B* and *C*, whose chances of solving it are 1/2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently? 7

(b) If Your squares are chosen at random on a chessboard, find the chance that they should be in a diagonal line. 8

4. *(a)* Let X be a continuous random variable with pdf,

$$f(x) = \begin{cases} ax, & 0 \quad x \quad 1 \\ a, & 1 \quad x \quad 2 \\ ax \quad 3a, & 2 \quad x \quad 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Determine the constant a.

(ii) $P(X \quad 1 \quad 5)$ and $P(1 \quad X \quad 2 \quad 5)$. 3+4=7

(b) Define arithmetic mean, harmonic mean and geometric mean of a pdf $f(x)$. Prove that the geometric mean G of the pdf

$$F(x) \quad 6(2 \quad x)(x \quad 1), \; 1 \quad x \quad 2$$

is given by $6\log(16G) \quad 19$. 3+5=8

UNIT—III

5. *(a)* Define joint probability mass function, marginal probability function and conditional probability function. If X and Y are two random variables having the joint density function $f(x, y) \quad \frac{1}{27}(2x \quad y)$, where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X \quad x$. 10

(b) A continuous random variable X has the pdf, $f(x) \quad A \quad Bx$, $0 \quad x \quad 1$. If the mean of the distribution $\frac{1}{2}$, find A and B. 5

6. *(a)* Define mathematical expectation. A coin is tossed until a head appears. What is the expectation of the number of tosses required? 2+6=8

(b) In four tosses of a coin let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. By simple counting, derive the distribution of X and hence calculate the expected value of X. 7

UNIT—IV

7. *(a)* Define binomial distribution. In a precision bombing attack, there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? 2+6=8

(b) Define Poisson distribution. Show that Poisson distribution is a limiting case of binomial distribution. 2+5=7

8. *(a)* If X is a Poisson variate such that

$$P(X \quad 2) \quad 9P(X \quad 4) \quad 90P(X \quad 6)$$

Find—

(i) , the mean of X and

(ii) , the coefficient of skewness 5

(b) Six coins are tossed 6400 times. Using Poisson distribution, find the probability of getting six heads 50 times. 5

(c) A manufacturer of pins knows that 5% of his product is defective. If he sells pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality? 5

UNIT—V

9. *(a)* If X and Y are random variables and

$$r(aX \quad bY, bX \quad aY) \quad \frac{1 \quad 2ab}{a^2 \quad b^2}$$

find $r(X, Y)$, the coefficient of correlation between X and Y. 7

(b) The random variables X and Y have the joint pdf,

$$f(x, y) \quad \frac{1}{3}(x \quad y), 0 \quad x \quad 1, 0 \quad y \quad z$$

Find—

(i) $r(X, Y)$, the correlation coefficient between X and Y;

(ii) the two lines of regression;

(iii) the two regression curves for the means 2+2+4=8

10. *(a)* The variables X and Y are connected by the equation $aX \quad bY \quad C \quad 0$. Show that the correlation between them is –1, if the signs of a and b are alike and +1, if they are different. 5

(b) Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y) : 5

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

(c) The joint probability distribution of X and Y is given below : 5

X	–1	+1
Y		
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient between X and Y.

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