UG Even Semester (CBCS) Exam., May—2017

COMPUTER SCIENCE

(4th Semester)

Course No. : MCS–404 (C)

(Discrete Mathematics)

Full Marks : 75

Pass Marks : 30

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define converse and contrapositive of a statement. Write the converse and contrapositive of the following statements : 1+4=5

(i) If she works, she will earn money.

(ii) If it snows, then they do not drive the car.

(b) Show that $(p \quad (\quad p \quad q))$ and $p \quad q$, are logically equivalent. 5

(c) Write the logical forms of the following statements and then write the negation of each statement : 5

(i) "At least 10 inches of rainfall today in Delhi."

(ii) "If the teacher is absent, then some students do not complete their homework."

(iii) "All the students completed their homework and the teacher is present."

(iv) "Some of the students did not complete their homework or the teacher is absent."

2. *(a)* Determine the validity of the following arguments : 4+3+3=10

(i) If I study, then I will pass.
If I do not go to a movie, then I will study.

I failed.

Therefore, I went to a movie.

(ii)

$$\begin{array}{l} p \quad q \\ r \quad q \\ \quad r \\ \hline \quad p \end{array}$$

(iii)

$$\begin{array}{l} p \quad q \\ \quad q \\ \hline \quad p \end{array}$$

(b) Define universal quantifier and existential quantifier with examples. 5

UNIT—II

3. *(a)* Let G be the set of all 2×2 matrices

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

where a, b, c, d are real numbers, such that $ad \quad bc \quad 0$. Show that G is a group with respect to matrix multiplication. 6

(b) Let Q be the set of rational numbers and let be the operation on Q defined by $a \quad b \quad a \quad b \quad ab$. Show that Q is a group with respect to . 5

(c) Define semigroup. Let $S \quad N \quad N$ and be the operation on S defined by $(a, b) \quad (a\,, b\,) \quad (aa\,, bb\,)$. Show that $(S, \quad)$ is a semigroup. 4

4. *(a)* Define ring and integral domain. Show that every finite integral domain is a field. 2+5=7

(b) If H and K be any two subgroups of a group G, then show that HK is a subgroup of G if and only if $HK \quad KH$. 4

(c) Let $S \quad N \quad N$ and be the operation on S defined by

$$(a, b) \quad (a\,, b\,) \quad (a \quad a\,, b \quad b\,)$$

(i) Show that is associative.

(ii) Define $f : (S, \quad) \quad (z, \quad)$ by $f(a, b) \quad a \quad b$. Show that f is a homomorphism. 4

UNIT—III

5. *(a)* Define Hasse diagram. If D_m denotes the set of divisors of m ordered by divisibility, then draw the Hasse diagram of D_{36} and D_{48}. 1+2+2=5

(b) Define distributive lattice. Let L be a bounded distributive lattice. Then show that the complements are unique if they exist. 1+4=5

(c) Let L be a complemented lattice with unique complements. Then show that the join irreducible elements of L, other than O, are its atoms. 5

6. *(a)* Consider the lattice L in the figure given below :

(i) Which non-zero elements are join irreducible?

(ii) Which elements are atoms?

(iii) Is L distributive?

(iv) Find the complements, if they exist for the elements a, b and c.

(v) Which of the following are sub-lattices of L?

L_1 $\{O, a, b, I\}$, L_2 $\{O, a, e, I\}$

L_3 $\{a, c, d, I\}$, L_4 $\{O, c, d, I\}$ 5

(b) Let $f: z \quad z$ and $g: z \quad z$ are two functions defined as $f(x) \quad 2x \quad 3$ and $g(x) \quad 3x \quad 2$. Are the functions f and g one-one? Find $f \circ g(\ 5)$ and $g \circ f(3)$, where '$\circ$' represents the composition of functions. 5

(c) Out of 100 persons in a city, 80 take tea, 40 take coffee and 25 take both tea and coffee. How many persons take neither tea nor coffee? 5

UNIT—IV

7. *(a)* Define Boolean algebra. Show that the following are equivalent in a Boolean algebra : 2+4=6

(i) $a \quad b \quad b$

(ii) $a \quad b \quad a$

(iii) $a \quad b \quad 1$

(iv) $a \quad b \quad 0$

(b) Define sum of products form, complete sum of products form and prime implicants in Boolean algebra. 3

(c) Write an algorithm for finding sum of products form of a Boolean expression. 3

(d) Write an algorithm for finding prime implicants of a Boolean expression. 3

8. *(a)* Express each of the following Boolean expressions as a sum of products and then in its complete sum of products form : 5

(i) $E(x, y, z) \quad x(xy \quad x\,y \quad y\,z)$

(ii) $E(x, y, z) \quad z(x \quad y) \quad y$

(b) Find all the prime implicants of the following Boolean expression : 5

$E(x, y, z, t) \quad xy \quad y\,t \quad x\,yz \quad xy\,zt$

(c) If A 11001, B 01010 and C 11010, then find the output of the following circuit : 5

UNIT—V

9. *(a)* Define complete, regular and bipartite graphs with examples. 5

(b) Define planar and non-planar graphs. Show that complete graph of five vertices is non-planar. 5

(c) Find a minimum spanning tree of the following graph : 5

10. *(a)* Write the Dijkstra's algorithm for finding shortest path of a weighted graph between any two vertices. Using Dijkstra's algorithm, find the shortest path between the vertices a and z for the following graphs : 4+4+4=12

(i)

(ii)

(b) Define adjacency matrix. Draw the directed graph for the following adjacency matrix : 1+2=3

	v_1	v_2	v_3	v_4
v_1	1	0	1	0
v_2	1	0	0	1
v_3	0	1	0	1
v_4	1	0	1	0

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