UG Even Semester (CBCS) Exam., May—2017

COMPUTER SCIENCE

(2nd Semester)

Course No. : MCSCC–202

(Statistical Methods and Applications)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* A committee of four people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the following manner :

(i) There must be one from each category

(ii) It should have at least one from the purchase department

(iii) The chartered accountant must be in the committee 2+2+2=6

(b) Define conditional probability. State and prove multiplication law of probability. 5

(c) If two dice are thrown, what is the probability that the sum is neither 9 nor 10? 3

2. *(a)* Let A and B be two events, such that $P(A)$ $3/4$ and $P(B)$ $5/8$. Show that—

(i) $P(A \quad B) \quad \frac{3}{4}$

(ii) $\frac{3}{8} \quad P(A \quad B) \quad \frac{5}{8}$ 2+2=4

(b) The odds that person X speaks the truth are 3 : 2 and the odds that person Y speaks the truth are 5 : 3. In what percentage of cases are they likely to contradict each other on an identical point? 6

(c) For any three events A, B and C, prove that

(i) $P(A \quad B|C)$

$P(A|C) \quad P(B|C) \quad P(A \quad B|C)$

(ii) $P(A|C) \quad P(A \quad \overline{B}|C) \quad P(A \quad B|C)$ 4

UNIT—II

3. *(a)* The odds against manager X settling the wage dispute with the workers are 8 : 6 and odds in favour of manager Y settling the same dispute are 14 : 16.

(i) What is the chance that neither settles the dispute, if they both try independently?

(ii) What is the probability that the dispute will be settled? 5

(b) The diameter of an electric cable say X, is assumed to be a continuous random variable with p.d.f. $f(x)$ $6x(1 \quad x)$, $0 \quad x \quad 1$.

(i) Check that $f(x)$ is actually a p.d.f.

(ii) Determine a number b, such that

$$P(X \quad b) \quad P(X \quad b)$$ 5

(c) Prove that the geometric mean G of the distribution function,

$$F(x) \quad 6(2 \quad x)(x \quad 1), \quad 1 \quad x \quad 2$$

is given by $6\log(16G) \quad 19$. 4

4. *(a)* A continuous distribution of a variable X in the range (3, 3) is defined by

$$f(x) \quad \begin{cases} \frac{1}{16}(3 \quad x)^2, & 3 \quad x \quad 1 \\ \frac{1}{16}(6 \quad 2x^2), & 1 \quad x \quad 1 \\ \frac{1}{16}(3 \quad x)^2, & 1 \quad x \quad 3 \end{cases}$$

(i) Verify that the area under the curve is unity.

(ii) Find the mean and variance of the above distribution. 5

(b) A random variable X assumes the values 3, 2, 1, 0, 1, 2, 3 such that

$$P(X \quad 3) \quad P(X \quad 2) \quad P(X \quad 1),$$
$$P(X \quad 1) \quad P(X \quad 2) \quad P(X \quad 3) \text{ and}$$
$$P(X \quad 0) \quad P(X \quad 0) \quad P(X \quad 0)$$

Obtain probability mass function and distribution function of X. 4

(c) A continuous random variable X has a p.d.f., $f(x)$ $3x^2$, $0 \quad x \quad 1$. Find a and b such that

(i) $P\{X \quad a\} \quad P\{X \quad a\}$

(ii) $P\{X \quad b\} \quad 0\ 05$ 5

UNIT—III

5. *(a)* Let X and Y are two random variables having the joint density function,

$$f(x, y) \quad \frac{1}{27}(2x \quad y)$$

where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X \quad x$. 5

(b) In four tosses of a coin, let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. By simple counting, derive the distribution of X and hence calculate the expected value of X. 5

(c) A coin is tossed until a head appears. What is the expectation of the number of tosses required? 4

6. *(a)* If the possible values of a random variable X are 0, 1, 2, 3, ..., then show that

$$E(X) \quad \sum_{n \quad 0} P(X \quad n) \qquad 5$$

(b) Let X and Y be two random variables each taking three values 1, 0 and 1, and having the joint probability distribution :

X / Y	1	0	1	Total
1	0	0·1	0·1	0·2
0	0·2	0·2	0·2	0·6
1	0	0·1	0·1	0·2
Total	0·2	0·4	0·4	1·0

(i) Show that X and Y, have different expectations.

(ii) Prove that X and Y are uncorrelated.

(iii) Find variance of X and variance of Y.

(iv) Given that Y 0, what is the conditional probability distribution of X? 5

(c) Define mathematical expectation. Show that $E(X \quad Y) \quad E(X) \quad E(Y)$, where X and Y are random variables. 4

UNIT—IV

7. *(a)* Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. 4

(b) In a precision bombing attack, there is a 50% chance that any one bomb

will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? 5

(c) Find first, second and third moments about origin for binomial distribution. 5

8. *(a)* Show that Poisson distribution is a limiting case of Binomial distribution. 6

(b) A manufacturer of pins knows that 5% of his product is defective. If he sells pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality? 4

(c) If X is a Poisson variate, such that

$$P(X \quad 2) \quad 9P(X \quad 4) \quad 90\,P(X \quad 6)$$

find—

(i) , the mean of X;

(ii) , the coefficient of skewness. 4

UNIT—V

9. *(a)* Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y) : 6

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

(b) Define correlation coefficient and mention its properties. 4

(c) The variables X and Y are connected by the equation $aX \quad bY \quad c \quad 0$. Show that the correlation between them is 1, if the signs of a and b are alike and 1 if they are different. 4

10. *(a)* If X and Y are random variables and

$$r(aX \quad bY, bX \quad aY) \quad \frac{1 \quad 2ab}{a^2 \quad b^2}$$

find $r(X, Y)$, the coefficient of correlation between X and Y. 6

(b) Write down different properties of regression coefficients. 4

(c) The joint probability distribution of X and Y is given below :

X / Y	1	1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient between X and Y. 4

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