UG Even Semester (CBCS) Exam., May—2017

COMPUTER SCIENCE

(4th Semester)

Course No. : MCSCC–402

(Discrete Mathematics)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define converse and contrapositive of a statement. Write the converse and contrapositive of the following statements : 1+4=5

(i) If she works, she will earn money.

(ii) If it snows then they do not drive the car.

(b) Show that, (P (P q)) and P q are logically equivalent. 4

(c) Write the logical terms of the following statements and then write the negation of each statement : 5

(i) "At least 10 inches of rain fall today in Delhi."

(ii) "If the teacher is absent, then some students do not complete their homework."

(iii) "All the students completed their homework and the teacher is present."

(iv) "Some of the students did not complete their homework or the teacher is absent."

2. *(a)* Determine the validity of the following arguments : 4+3+3=10

(i) If I study, then I will pass
If I do not go to a movie, then I will study
I failed
———————————————
Therefore, I went to a movie

(ii) P q
r q
———————
r
———
P

(iii) P q
———
q
———
P

(b) Define universal quantifier and existential quantifier with examples. 4

UNIT—II

3. *(a)* Let G be the set of all 2 2 matrices,

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

where a, b, c and d are real numbers, such that ad bc 0. Show that G is a group w.r.t. matrix multiplication. 6

(b) Let Q be the set of rational numbers and let be the operation Q, defined by, a b a b ab. Show that Q is a group w.r.t. . 4

(c) Define semi-group. Let S N N and be the operation on S defined by (a, b) (a, b) (aa, bb). Show that $(S,$ $)$ is a semi-group. 4

4. *(a)* Define ring and integral domain. Show that every finite integral domain is a field. 2+4=6

(b) If H and K be any two subgroups of a group G, then show that HK is a subgroup of G if and only if HK KH. 4

(c) Let S N N and be the operation on S defined by

(a, b) (a, b) $(a$ a, b $b)$

(i) Show that is associative.

(ii) Define $f : (S,$ $)$ $(Z,$ $)$ by

$f(a, b)$ a b

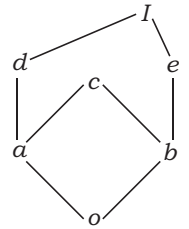
Show that f is a homomorphism. 4

UNIT—III

5. *(a)* Define Hasse diagram. If D_m denotes the set of divisors of m ordered by divisibility, then draw the Hasse diagram of D_{36} and D_{48}. 1+2+2=5

(b) Define distributive lattice. Let L be a bounded distributive lattice, then show that complements are unique if they exist. 5

(c) Consider the lattice L in the figure given below :



(i) Which non-zero elements are join irreducible?

(ii) Which elements are atoms?

(iii) Is L distributive?

(iv) Find the complements, if they exist for the elements a, b and c. 4

6. *(a)* Define Boolean algebra. Show that the following are equivalent in a Boolean algebra : 2+4=6

(i) a b b *(ii)* a b a

(iii) a b 1 *(iv)* a b 0

(b) Express each of the following Boolean expression as a sum of products and then in its complete sum of products form : 4

(i) $E(x, y, z)$ $x(xy$ $x\,y$ $y\,z)$

(ii) $E(x, y, z)$ $z(x$ $y)$ y

(c) Find all the prime implicants of the following Boolean expression : 4

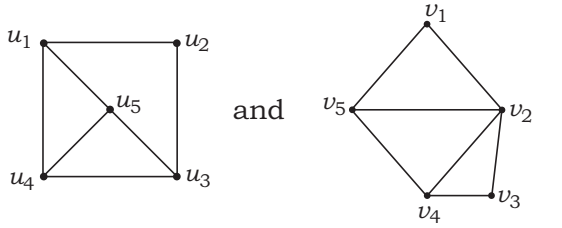
$E(x, y, z, t)$ xy $y\,t$ $x\,yz$ $xy\,zt$

UNIT—IV

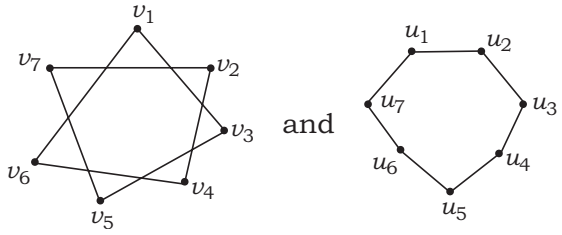
7. *(a)* Define complete, regular and bipartite graphs with examples. 5

(b) Define isomorphic graphs. Check whether the following pairs of graphs are isomorphic or not : 1+4=5

(i)



(ii)



(c) Define path, circuit, degree of a vertex, connected graph, Euler path and Hamiltonian circuit in a graph. 4

8. *(a)* Define incidence matrix and adjacency matrix for a graph. Draw the graphs for the following adjacency matrix : 2+(2+2)=6

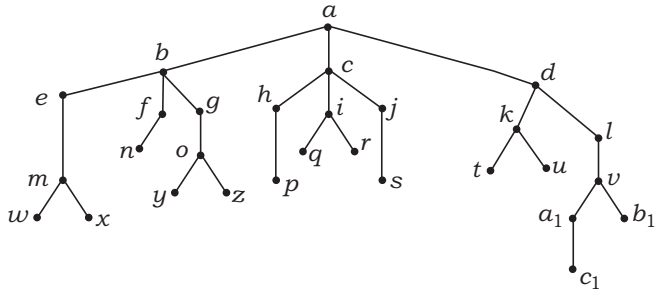
(i)

0	2	3	0
1	2	2	1
2	1	1	0
1	0	0	2

(ii)

0	3	0	2
3	0	1	1
0	1	1	2
2	1	2	0

(b) Write the preorder, inorder and postorder traversal algorithm for a rooted tree. For the following rooted tree, find the order in which the vertices of the tree will be visited by postorder traversal and preorder traversal : 3+2½+2½=8

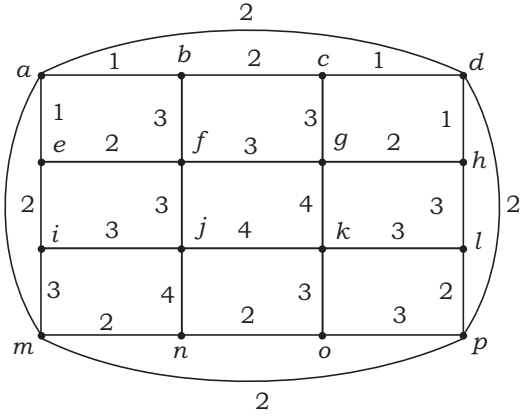


UNIT—V

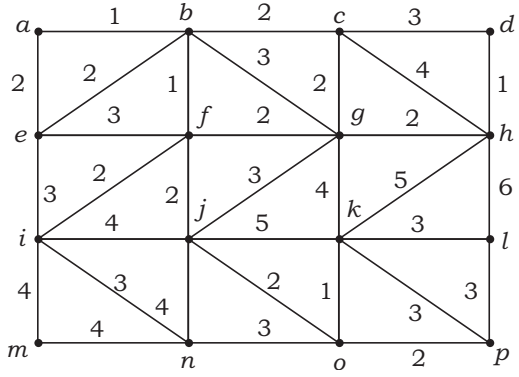
9. *(a)* Define spanning tree and minimum spanning tree. Write the algorithm for finding minimum spanning tree of a weighted graph. 2+3=5

(b) By using Kruskal algorithm, find a minimum spanning tree for the following graphs : 5+4=9

(i)

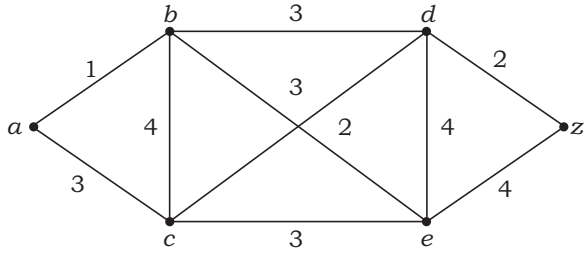


(ii)

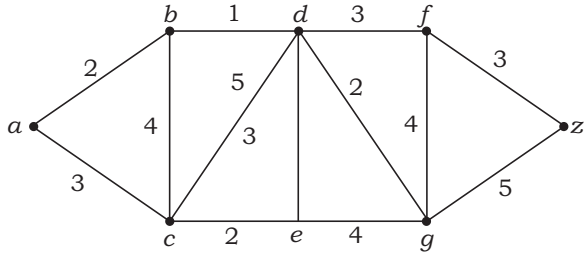


10. Write Dijkstra's algorithm for finding the shortest path of a weighted graph. Using Dijkstra's algorithm, find the shortest path between the vertices *a* and *z* for the following graphs : 4+3+4+3=14

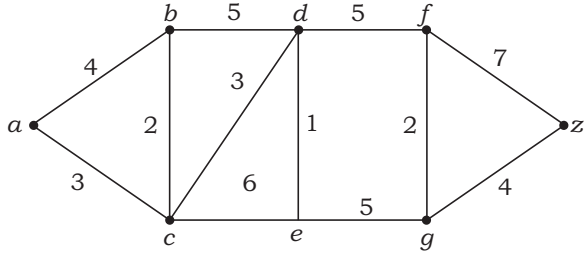
(a)



(b)



(c)



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