UNIT - IV

7. a) Find the roots of $x^2 - 4x - 10 = 0$ using Newton-Raphson's method correct to three decimal places. 6

   b) Find an approximate value of y (0.2) by solving the differential equation $\frac{dy}{dx} = x + y^2$

   with the initial condition $y = 1$ when $x = 0$, using fourth order Runge-Kutta method. 8

8. a) Obtain the value of the definite integral

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x)\, dx$$

   using Simpson's rule. 7

   b) Solve the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y_0 = 1$ when $x_0 = 1$, using Picard's method of successive approximation. 7

UNIT - V

9. a) Express $\vec{\nabla}$ x $\vec{A}$ in orthogonal co-ordinates. 6

   b) Express velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical co-ordinates. 4+4=8

10. a) Obtain the expression for $\nabla^2\psi$ in orthogonal curvi linear co-ordinates and hence express $\nabla^2\psi$ in spherical co-ordinates. 5+2=7

   b) Define Christoffel's symbols of 1st and 2nd kind. Show that $\frac{\partial g_{ik}}{\partial x^j} = [i\,j\,,k] + [k\,j\,,i]$ 2+3=5

   c) Define covariant derivative of a tensor. 2

PG (CBCS) ODD SEMESTER EXAMINATION, 2021
Held in July 2022

PHYSICS
1st Semester

Course No. : PHYCC - 102
(Mathematical Physics I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions, taking one question from each unit)

UNIT - I

1. a) Define inner product of two vectors. Show that inner product is anti linear with respect to the pre factor. 2+2=4

   b) Prove the Cauchy-Schwartz inequality. 4

   c) Show that the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

   are (i) Hermitian, (ii) form an orthonormal set of vectors and (iii) anticommute with each other. 1+3+2=6

P.T.O.

2. a) From the set of vectors $\{f_1(x) = 1, f_2(x) = x, f_3(x) = x^2, f_4(x) = x^3\}$, construct an orthonormal set using Gram-Schmidt procedure. 6

b) Verify Cayley-Hamilton theorem for the matrix 5

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{pmatrix}$$

c) Show that under similarity transformation $A \rightarrow SAS^{-1}$, the set of eigen values of the similar operatos remain unaltered. 3

UNIT - II

3. a) Determine whether $f(z) = z^{-1}$ is analytic. 3

b) Show that $u(x, y) = e^{x^2 - y^2}\cos(xy)$ is a harmonic function. Obtain its harmonic conjugate. 2+3=5

c) State Cauchy's integral theorem. 2

d) If a function $f(z)$ is analytic at the point $z = z_0$, show that its n^{th} derivative at $z = z_0$ is given by

$$f^{(n)}(Z_0) = \frac{n!}{2\pi!} \oint_c \frac{f(z)}{(Z - Z_0)^{n+1}}\, dz$$

where C is a closed contour enclosing the point $z = z_0$ 4

4. a) Obtain the series expansion of the function 4

$$f(z) = \frac{1}{1 - z}$$

about z = 0

b) Show that the residue of the function $f(z)$ at a pole of order m at $z = z_0$ is given by 4

$$R = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)]$$

c) State Cauchy's residue theorem. Evaluate the definite integral using residue theorem. 2+4=6

$$\int_0^\infty \frac{dx}{1 + x^6}$$

UNIT - III

5. a) Expand the function $f(x) = x + x^2$ where $-\pi \le x \le \pi$, in a Fourier series and hence show that $\sigma(z) = \frac{\pi^2}{6}$, where $\sigma(k) = \sum_{n=1}^{\infty} \frac{1}{n^k}$ is the Reimann Zeta function. 6+3=9

b) Find the Fourier transform for the function 5
$f(x) = 1 - x^2$, $-1 \le x \le 1$

6. a) A function $f(x)$ of period 2π is given by

$f(x) = 0$ for $-\pi < x < 0$

$= x$ for $0 < x < \pi$

i) Sketch a graph of the function f(x) for the interval $-3\pi < x < 3\pi$ 2

ii) Obtain the Fourier expansion of the function. 5

iii) Show that 3

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots$$

b) Obtain the Laplace transforms of sin(kt) and cos(kt). 2+2=4