d) Consider the following two sets: 3

$$|\psi\rangle = \begin{pmatrix} 5i \\ 2 \\ -i \end{pmatrix}, \qquad |\phi\rangle = \begin{pmatrix} 3 \\ 8i \\ -9i \end{pmatrix}$$

Find (i) $\langle\psi|$ and $|\psi\rangle^*$
(ii) Are $|\psi\rangle$ and $|\phi\rangle$ orthogonal.

8. a) What do you mean by unitary transformation? Describe briefly the infinitesimal and finite unitary transformations. 7

b) Calculate the commutator $[\hat{x}, \hat{p}]$ in the position and momentum representations and show that the commutator $[\hat{x}, \hat{p}]$ is representation independent. 7

UNIT - V

9. Consider the case where angular momentum, j = 1

a) Find the matrices representing the operators $\hat{J}^2$, $\hat{J}_z$, $\hat{J}_\pm$, $\hat{J}_x$ & $\hat{J}_y$ 6

b) Find the joint eigenstates of $\hat{J}^2$ and $\hat{J}_z$ and verfity that they form an orthonormal and complete basis. 8

10. a) What are Clebsch-Gorden coefficients? Show that these coefficients vanish unless $m_1+m_2 = m$. 7

b) Consider two commuting angular momentum operators $\hat{J}_1$ and $\hat{J}_2$. If their sum is denoted by $\hat{J}$ i.e., $\hat{J} = \hat{J}_1 + \hat{J}_2$, show that $\hat{J}^2$, $\hat{J}_z$, $\hat{J}_1^2$ and $\hat{J}_2^2$ form a complete set of commuting operators. 7

PG (CBCS) ODD SEMESTER EXAMINATION, 2021
Held in July 2022

PHYSICS
1st Semester

Course No. : PHYCC - 103
(Quantum Mechanics-I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions, taking one question from each unit)

UNIT - I

1. a) Evaluate the ratio of the de Broglie wavelength of electron to that of proton when (i) both have the same kinetic energy (ii) the kinetic energies of the electron and proton are 1000 and 100 ev, respectively. 4

b) If the stopping potential of a metal when illuminated with a radiation of wavelength 150nm is 7.5 V, calculate the stopping potential of the metal when illuminated by a radition of wavelength of 275 nm. 5

c) Show that the maximum kinetic energy (K.E.) transferred to a proton when hit by a photon of energy hv is given by

$$K.E. = h\upsilon / \left[1 + \frac{m_p c^2}{2h\upsilon}\right]$$

where $m_p \rightarrow$ mass of proton. 5

2. a) State and provle Ehrenfest's theorem. 7

b) A bullet of mass 0.03 g is moving with a velocity of 500 m/s. The speed is measured up to an accuracy of 0.02%. Calculate the uncertainty in its position. Also comment on the result. 3

c) Consider two states

$|\psi_1> = |\phi_1> + 4i|\phi_2 + 5|\phi_3>$

and

$|\psi_2> = b|\phi_1> + 4|\phi_2> -3i|\phi_3>$

where $|\phi_1>$, $|\phi_2>$ and $|\phi_3$ are orthonormal kets, and b is a constant. Find the value of b so that $|\psi_1>$ and $|\psi_2>$ are orthogonal. 4

UNIT - II

3. a) A stream of particles of mass m and energy E moves towards the potential step $v(x) = 0$ for $x<0$ and $v(x) = v_0$ for $x>0$. If the energy of the particles $E>V_0$, then show that the sum of fluxes of the transmitted and reflected particles is equal to the flux of incident particles. 10

b) What do you mean by stationary states? Show that the expectation value of an observable whose operator does not depend on time explicity is a constant. 4

4. a) A beam of 12 ev electrons is incident on a potential barrier of height 30 eV and width 0.05 nm. Calculate the transmission coefficient. 4

b) Find the energy eigenvalues and eigenfunctions of a particle of mass m moving in a one dimensional potential specified by 10

$$v(x) = \begin{cases} 0, & -a < x < a \\ \infty, & otherwise \end{cases}$$

UNIT - III

5. a) Compare the Schroedinger, Heisenberg and Dirac interaction pictures.

Show that the time evolution for operator $(\hat{A}_H)$ in the Heisenberg picture is given by 3+6=9

$$\frac{d\hat{A}_H}{dt} = \frac{\partial \hat{A}_H}{\partial t} + \frac{1}{it}\,[\hat{A}_H, \hat{H}]$$

b) Show that the commutator 5

$$\left[\hat{x}, [\hat{x}, \hat{H}]\right] = -\frac{\hbar^2}{m}$$

where $\hat{H}$ is the Hameltonian operator.

6. a) Use the uncertainty principle to estimate the size and energy of the ground state of hydrogen atom. 9

b) Obtain the energy-time uncertainty principle and interpret the quantities ΔE & Δt in it. 5

UNIT - IV

7. a) State and prove Parseval's theorem. 3

b) Write the orthonormality and completeness conditions in the discrete and continuous bases. 2

c) Obtain the expressions for the momentum and Hamiltonian operators in the position representation. 6