UG/PG (CBCS) ODD SEMESTER EXAMINATION, 2022

COMPUTER SCIENCE

3rd Semester

Course No. : MCSC-302

(Mathematics-II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT-I

1. (a) If f: A→R and if 'C' is a cluster point of 'A', then show that 'f' can have only one limit at 'C'. 5

   (b) Define limit of a function at any point 'C'. By using definition of limit, find the following limit. 5

   $$\lim_{x\to 2} \frac{x^3-4}{x^2+1}$$

   (c) Evaluate: 2+2=4

   (i) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-4x+3}$

   (ii) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1-3x}}{x}$

2. (a) Show that $\lim_{x\to\alpha}(x-\sqrt{x^2+x}) = \frac{1}{2}(a+b)$. 5

(b) If $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} \phi(x)= l'$, are finite quantities, then show that 6

(i) $\lim_{x\to a}\{f(x) + \phi(x)\}= l+l'$

(ii) $\lim_{x\to a}\{f(x) \text{ x } \phi(x)\}= ll'$

(c) Evaluate, $\lim_{x\to 0}\frac{\text{Sinx} - \text{tanx}}{x^3}$ 3

UNIT-II

3. (a) Define continuous function. By using $\in$-δ definition, show that the function f(x)= x is a continuous function. 5

(b) Show that the function, f(x)= x|+|x–1|+|x–2| is continuous at the points x=0 and x=1. 6

(c) Discuss the contuity of the following function at x=0. 3

$$f(x) = \begin{cases} \frac{x^4+4x^3+2x}{\sin x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$$

4. (a) If $y= \cos(m\sin^{-1}x)$, show that 6

(i) $(1-x^2)y_2-xy, + x^2y=0$

(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y=0$

(c) Prove that the series 5

$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots$ is convergent.

10. (a) Write limit comparison test, Ratio test and root test. 6

(b) Examine whether the following series are convergent or divergent :

(i) $x+\frac{1}{2}\cdot\frac{x^3}{3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\cdot\frac{x^7}{7} + \ldots.$

(ii) $\frac{1}{2} + \frac{2^2}{2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \ldots\ldots + \frac{n}{2^n} + \ldots\ldots$

4+4=8

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(b) Find for what values of x the following expression is maximum and minimum respectively : 3+3=6

(i) $2x^3 - 21x^2 + 36x - 20$

(ii) $5x^6 - 18x^5 + 15x^4 - 10$

(c) State Leibnitz's theorem, for n-th derivative of the product of two functions. 2

UNIT-III

5. (a) Define derivative of a function at any point 'C'. If $f:I \to R$, has a derivative at $C \in I$, then show that f is continuous at 'C'. 2+4=6

(b) By using $\in$-δ definition of derivative, prove the following identifies : 3+3+2=8

(i) $\frac{d}{dx}(\tan x) = \sec^2 x$

(ii) $\frac{d}{dx}(\phi(x).\psi(x)) = \phi(x)\psi'(x) + \psi(x).\phi'(x)$

(iii) $\frac{d}{dx}(\cos x) = -\sin x$

6. (a) Show that the function f(x), defined by : 6

$$f(x) = \begin{cases} 3+2x, & -3/2 < x < 0 \\ 3-2x, & 0 < x \leq 3/2 \end{cases}$$

is continuous but not differentiable at x=0.

(b) Let $f(x) = \begin{cases} x^2, & \text{when x is rational} \\ 0, & \text{when x is irrational} \end{cases}$ 3

show that $f'(O) = 0$.

(c) If $f(a)=2$, $f'(a)=1$, $g(a)=-1$, and $g'(a)=2$, then find the value of 5

$$\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x-a}$$

UNIT-IV

7. (a) By using L' Hospital's theorem, Evaluate the following limits : 3+3=6

(i) $\lim_{x \to 1} \left\{\frac{x}{x-1} - \frac{1}{\log x}\right\}$

(ii) $\lim_{x \to 0} \left\{\frac{1}{x} - \frac{2}{x(e^{x+1})}\right\}$

(b) State and prove, Mean Value theorem. 2+4=6

(c) Define convergent sequence. 2

8. (a) Show that every convergent sequence of real numbers is bounded. 4

(b) Define Cauchy sequence. If $x=(x_n)$ is a convergent sequence of real numbers, then show that x is a Cauchy sequence. 5

(c) Let (x_n) and (y_n) be sequences of real numbers, that converges to x and y respectively. Show that the sequence $(x_n + y_n)$ and (x_n, y_n) converges to x+y and x, y respectively. 5

UNIT-V

9. (a) If $x_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)}$,

then show that (xn) is a bounded monotomic increasing sequences. 5

(b) Define bounded sequence and monotomic sequence with example. 2+2=4