UG/PG (CBCS) ODD SEMESTER EXAMINATION, 2022

COMPUTER SCIENCE

1st Semester

Course No. : MCSC-102

(Mathematics-I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT-I

1. (a) If A, B and C are any three monempty sets then prove the following : 2+2+3=7

(i) $A - (B \cap C) = (A-B) \cup (A-C)$

(ii) $A\ (B \cap C) = (A \cup B) \cap (A \cup C)$

(iii) $A \times (B \cup C) = (A \times B)\ (A \times C)$

(b) Define equivalence relation. If R be a relation in the set of integers Z, defined by the open sentence (x–y) is divisible by 6, that is,

R– {(x,y) : $x \in Z$, $y \in Z$, (x–y) is divisible by 6}, then prove that R is an equivalance relation. 7

2. (a) Define one-one and onto function. If f:A→B be one-one and onto function, then prove that the inverse function of 'f' is unique. 7

(b) Show that the function :

(i) f: Q→Q defined by f(x)= 3x+2, is one-one and onto, where Q, is the set of rational numbers.

(ii) f: R→R, defined by f(x)= 3x+4, is both one-one and onto, where R is the set of real numbers. 7

UNIT-II

3. (a) Define rank of a matrix. Find the rank of the following matrix : 5

$$\begin{pmatrix} 2 & 4 & 6 & 8 \\ 1 & 2 & 3 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

(b) Find the inverse of the following matrices by using Gauss-Jordan method. 3+3=6

$$\begin{pmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix}$$

(c) Define eigen value and egen vector of a non-zero square matrix. 3

4. (a) Solve by Gauss-Elemination method : 5

$3x + 4y + 5z = 18$

$2x - y + 8z = 13$

$5x - 2y + 4z = 20$

(b) State and prove shifting property in Laplace Transformation. 4

(c) Find the Laplace Transformation f(t), defined as- 4

$$f(t) = \begin{cases} \text{Sint}, & 0<t<\pi \\ 0, & t>\pi \end{cases}$$

10. (a) Find the inverse Laplace Transformation of - 3+3+6

(i) $\frac{S+2}{S^2-4S+13}$ (ii) $\frac{2S-3}{S^2-S-3/4}$

(b) Solve the following equations by using Laplace Transformation : 4+4=8

(i) $y'' + 4y' + 8y = \cos 2t$,

$y(0) = 2$ and $y'(0)=1$

(ii) $y''+4y= 4t$, that satisfies the initial condition $y(0)=1$ and $y'(0)=5$

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(b) Find eigen values and eigen vectors of the following matrix : 5

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

(c) By using Cayley-Hamilton theorem, find the inverse of the following matrix. 4

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

UNIT-III

5. (a) Define scalar product and craxs product of two vectors with example. 4

(b) Find the angle between the following pair and vectors : 2+2=4

(i) $4\hat{i} - 2\hat{j} - 2\hat{k}$, $3\hat{i} + 4\hat{j} - 4\hat{k}$

(ii) $-\hat{i} + 2\hat{j} - 3\hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$

(c) If $\vec{F} = x^2z\hat{i} + 2x^2y^2z^2\hat{j} - 4yz^2\hat{k}$, find div($\vec{F}$) and Chrl($\vec{F}$) at the point (1, –1, 1). 6

6. (a) Compute the following vector products :

(i) $[(\hat{i} - 2\hat{j} + \hat{k}) \times (\hat{i} - 3\hat{j} + \hat{k})] \times [(2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} + 2\hat{j} - 3\hat{k})]$

(ii) $[(\hat{i} + \hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})] \times [(\hat{i} - \hat{j} + 2\hat{k}) \times (2\hat{j} - 3\hat{k})]$ 4+4=8

(b) A particle moves along the curve

$x + t^3 + 1$, $y = t^2$ and $z = 2t+5$, where it is the time. Find the components of its velocity and

acceleration at time t= 1 in the direction $\hat{i} + \hat{j} + 3\hat{k}$. 6

UNIT-IV

7. (a) Solve the following differential equations : 3½+3½=7

(i) $(x^2 - 2y^2)dx + xy.dy=0$

(ii) $(Sinxtany + 1)dx - cosx\ sec^2ydy=0$

(b) Solve the following differential equations : 3+2+2=7

(i) $(3x^2 - y^2)dy - 2xydx =0$

(ii) $(1+x^2)dy + 2xydx = cotxdx$

(iii) $\frac{dy}{dx} + ycotx = 2xcosecx$

8. (a) Find th solution of the following initial value problems : 3+3+3=9

(i) $y'' - 5y' + 5y = 0$, $y(1)= e^2$ and $y'(1)= 30^2$

(ii) $y'' - 6y' + 96= 0$, $y(0)= 0$ and $y'(0)= 11$

(iii) $y'' + 8y' - 9y=0$, $y(1)= 2$ and $y'(1)= 0$

(b) By using method of undtermined coefficients find the general solution of the following equation : 5

$y'' - 2y' + 5y = 25x^2 + 12$

UNIT-V

9. (a) Find the Laplace Transform of the following : 3+3=6

(i) $te^{-4t}Sin3t$ (ii) $Sin2tSin3t$