(c) The magnetic potential V for a circular disc of radius a and strength w, magnetised parallel to its axis, satisfies Laplace's equation. V equals to $2\pi w$ on the disc itself and vanishes at exteriar points in the plane of the disc. Show that at the point (r, z), z > 0 8

$$V = 2\pi\omega a \int_0^{\infty} e^{sz} J_0(rs) J_1(as)\, ds$$

★★★

PG (CBCS) ODD SEMESTER EXAMINATION, 2022

MATHEMATICS

3rd Semester

Course No. : MTMCC - 301

(Integral Equations and Calculus of Variations)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) Show that the curves $y = \frac{\sin x}{x}$, $y_1 = 0$ on [0, π] when n is sufficiently large are close in the sense of proximity of any order. 5

(b) Find the first order distance betweeen the curves $f(x) = lnx$ and $f_1(x) = x$ on the interval $[e^{-1}, e]$ 4

(c) Discuss the continuity of the functional $J[y(x)] = \int_a^b y^2(x)\, dx$, where $y(x) \in C_1[0, \pi]$ and x_0

(Turn Over)

$\in [0, \pi]$ on the function $y(x) = 0$ in the sense of first order proximity. 5

2. (a) Show that the functional $J[y(x) = \int_a^b y^2(x)\,dx$ defined in the space C [a, b] is differentiable at every point $y(x)$. 4

(b) Find the extremal of $J[y(x)] = \int_0^1 [y'(x)]^2\,dx$, $y(0)=1$, $y(1) = 6$ subject to $\int_0^1 y(x)dx = 3$ 5

(c) Derive the Euler-Poisson equation for the function of the form $F(x, y, y^1, \ldots\, y^{(n)})$ 5

UNIT - II

3. (a) Solve the following integral equation by successive approximation method. 4

$$u(x) = 1 + \int_0^1 x\,u(t)dt$$

(b) Using Adomian decomposition method, solve

$$u(x) = e^x - 1 + \int_0^1 t\,u(t)dt$$ 4

(c) Derive the resolvent kernel for the Fredhalm integral equation of second kind. 6

(ii) Find the Fourier sine and casine transform of 3

$$f(x) = \begin{cases} 1, & 0 \le x \le a \\ 0, & x > a \end{cases}$$

UNIT - V

9. (a) Find the Hankel transform of $x^2 e^{-x}$ or order one. 2

(b) Evaluate inverse Hankel transform of specified order :

(i) $H^{-1}\ \{e^{-as},\ n = 0\}$ 2

(ii) $H^{-1} = \left\{ \frac{e^{-as}}{s}\ ,\ n = 0 \right.$ 2

(c) Find the Hankel transform of 8

$$f(x) = \begin{cases} a^2 - x^2, & 0 < x < a\ ,\ n = 0 \\ 0, & x > a\ ,\ n = 0 \end{cases}$$

10. (a) If $f(x) = \begin{cases} 1, & 0 < x < a\ ,\ n = 0 \\ 0, & x > a\ ,\ n = 0 \end{cases}$ 3

show that $H\{f(x), n = 0\} = \frac{a}{s} J_1(sa)$

(b) Find the Hankel transform of $\frac{df}{dx}$ when $f(x) = \frac{e^{-ax}}{x}$ and $n = 1$ 3

(b) Find the solution of 8

$$x\,e^x - \int_0^x e^{x-t}\,u(t)\,dt$$, using any appropriate method.

UNIT - IV

7. (a) Find the Laplace transform of

(i) $f(t) = 4\,t\sin t\cos t + 2e^{t}$ 3

(ii) $f(t) = \sin \omega t \quad , \quad 0 < t < \frac{\pi}{\omega}$ 4

$\quad = 0 \quad , \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$

(b) Using Laplace transform solve : 7

$$\frac{dx}{dt} + \frac{dy}{dt} + x + y = 1$$

$$\frac{dy}{dt} = 2x + y$$

given $x = 0$ and $y = 0$ when t = 0

8. (a) Find the inverse Laplace transform of $\frac{1}{(p^2+q^2)^2}$ using convolution theorem, whre a is an arbitrary constant.

(b) (i) Find the Fourier transform of 3

$$f(x) = e^{-x^2/2}$$

4. (a) Convert the following IVP into correponding integral equation 7

$$y'''(x) - 3y'''(x) - 6y'(x) + 5y(x) = 0$$

subject to the conditions

$$y(0) = y'(0) = y''(0) = 1$$

(b) Solve by finite difference approximation method.

$$u(x) = 1 + \lambda \int_0^1 x\,t\,u(t)dt$$ 7

UNIT - III

5. (a) Derive solution of 8

$$u(x) = f(x) + \lambda \int_0^x k(x,t)\,u(t)\,dt$$

using method of successive approximation.

(b) Solve by successive substitution method 6

$$u(x) = x - \int_0^x (x-t)\,u(t)\,dt$$

6. (a) Using series solution method solve 6

$$u(x) = 1 + \int_0^x u(t)\,dt$$