# PG (CBCS) ODD SEMESTER EXAMINATION, 2022

## MATHEMATICS

3rd Semester

Course No. : MTMCC - 303

**( Mechanics )**

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

(Answer any five questions, taking one from each unit)

UNIT - I

1. (a) What do you mean by generalized co-ordinates? Establish that, under Lagrangian mechanics and in absence of generalized forces

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_j}\right) - \frac{\partial T}{\partial q_j} = 0$$

where the symbols used have their usual meaning. 1+6

(b) Show that Euler-Lagrange equations of motion are invariant under a generalized Co-ordinate transformations. Hence, obtain the equation of motion for a particle of mass 'm' moving under gravity along a plane curve $xy$ = c, (c > 0, a constrant) 5+2

2. (a) What do you mean by ignorable co-ordinates? Show that is a conservative system, the generalized momenta corresponding to any ignorable co-ordinates are constant. 1+2

(b) What is Hamilton's action principle? Use it to obtain the dynamical equations of motion of a particle moving on a cylindrical surface. 1+6

(c) A bead slides without friction on a frictionless wire in the shape of a cycloid with equation:

$x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ $0 \le \theta \le 2\pi$

Find the Lagrangian function. 4

## UNIT - II

3. (a) Establish that if a Lagrangian of a holonomic system is not unique so also the corresponding Hamiltonian. 5

(b) If $q_j$ (for some $j \neq j_0$) is a cyclic co-ordinate of a system then show that, $\dot{p}_{j_0} = 0 \ \ and \ \ \dot{q}_{j_0} = \frac{\partial H}{\partial P_{j_0}}$ although $\dot{p}_j \neq 0 \ for \ j \neq j_0$ 5

(c) For a 1-D system with the Hamiltonian given by $H(q,p) = \frac{p^2}{2} - \frac{1}{2q^2}$, show that there is a constant of motion expressed as, $x(p,q,t) = \frac{pq}{2} - Ht$ 4



## UNIT - V

9. (a) What is Galiban theory of Relativity? Show that under this theory, the space, time and acceleration quantities remains unchanged, while velocity is not an invariant. 9

(b) A rod of length 4.5 m lies in the *xy*-plane of stationary frame 's' and makes and angle of $60^0$ with y-axis. Find it's length and inclination with *x*-direction in frame S′ moving with a relative velocity of 0.8c along *x*-direction. 5

10. (a) State the postulates of Special Theory of Relativity (STR). What is time dilation? A clock keeps correct time. With what speed should it be moved relative to an observer so that it may seem to loose 1 minute in 24 hours? 2+1+3

(b) Wha tis length contraction hypothesis? The length of a spaceship is measured to be exactly half its proper length. What is the speed of the spaceship relative to the observer's frame. 1+2

(c) Two events separated by a distance 20m and time interval $3\times10^{-8}s$ observed in *s*-frame. Obtain the distance and time separation between the same two events from S′, moving relatively with speed $c/3$ along 20 *m* line. 5

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4. (a) Prove that a Hamiltonian function of a system represents the mechanical energy of a system & hence conserved, if the system is scleronomous and conservative. 6

(b) Obtain the position $x(t)$ and linear momentum $P_x(t)$ at any time '$t$' for the motion of a particle associated with a Hamiltonian function, given by $H(x, p_x) = \frac{px^2}{2m} + \frac{1}{2} mw^2x^2$ where, $m$ & $w$ are some constants. 8

## UNIT - III

5. (a) What is Hamilton-Jacobi (HJ) theorem? Show that the Hamilton Principal Function (HPF) is the complete integral of the corresponding HJ-equation. 2+6

(b) Find the cononical transformations through the solution of the HJ-equation, the generating function for which is $F_2 = S(q, p)$ transforms the Hamiltonian $H(q,p) = \frac{p^2}{2m} + \frac{mw^2q^2}{2}$ to K (Q, P) = $f(p)$ only. 6

6. (a) Establish that in case of a scleronomic system and under the Hamilton Characteristic Function (HCF) all the transformed linear momentum are constants, while one but all the generalized co-ordinates are constants. 8





(b) A particle of mass 'm' moves in 1-D subject to the potential given as $V(x) = -a\,cosec^2(x/x_0)$. Constant the HJ - equation and obtain the integral form of the HPF. 6

## UNIT - IV

7. (a) Establish that for a force free motion of a rigid body under rotation, the rotational kinetic energy of the body is always conserved. 8

(b) For a force free motion of a rigid body symmetrical about $y$ - principal axis with angular velocity $\vec{w} = (w_x, w_y, w_z)$ then provle that, $w_y$ = constant and that $w_x^2 + w_z^2 = constant$. 6

8. (a) If $\vec{L} = (L_1, L_2, L_3)$ and $I = (I_1, I_2, I_3)$ be the components of angular momentum and moment of inertia respectively of a rigid body rotation with no external torque, then establish the following results : 4+2

(i) $\frac{L_1^2}{I_1} + \frac{L_2^2}{I_2} + \frac{L_3^2}{I_3} = conserved$

(ii) $L_1^2 + L_2^2 + L_3^2 = conserved$

(b) A body moves uner no force about a fixed point 'o', the principal MI's at 'o' being 3k, 5k and 6k. Initially, the angular velocity of the body has components $w_1 = n$, $w_2 = o$, $w_3 = n$ about the principal axes. Show that at any time $w_2(t) = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right)$ 8