(b) Prove that a eyclic group is commutative but converse is not true. 4

(c) Define normal subgroup. Prove that every subgroup of a commutative group G is a normal in G. 1+3=4

(d) In a group (G, 0, a is an element of order 30. Find the order of a^{18}. 3

9. (a) Define commuative ring with example. Prove that $M_2(\mathbb{R})$ a ring with unity. 1+3=4

(b) Define devisor of zero in a ring. Give an example of a ring contains devisor of zero. 1+2=3

(c) Prove that a field is an integral domain but converse is not true. 4

(d) Prove that if a be a unit is a ring R with unity, then a is not a devisor of zero. 3

Or

10. (a) Let R be a ring with unity I, having no devisor of zero. Let a, b $\in$ R and ab=I. Prove that ba=I too. 4

(b) Prove that a Boolean ring is commutative. 3

(c) Prove that the ring of matrices

$$\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in Q\right\} \text{ is a field.}$$ 4

(d) Prove that cancellation law holds in a field. 3

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UG Odd Semester Examination, 2022

EDUCATION B.Sc. B.Ed

(3rd Semester)

Course No.: BSMP-303

(Classical algebra, Trigonometry, Modern and Linear Algebra)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

The figures in the right margin indicate full marks for the question.
Answer the following five (5) questions

1. (a) Solve the system of equations by matrix inversion method. 5

$$\begin{aligned} 2x + y + z &= 5 \\ x - y &= 0 \\ zx + y - z &= 1 \end{aligned}$$

(b) Obtain a row echelon matrix which is row equivalent to 5

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$

(c) Obtain the normal form of the matrix. 4

$$\begin{pmatrix} 2 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 1 \end{pmatrix}$$

Or

2. (a) Solve the equation $x^3+5x^2+7x+2=0$, give that the product of two of the roots is 1. 4

(b) Solve by Cardan's method 5

$x^3-9x+28=0$

(c) Use Sturm's functions to show that the roots of the equation are all real and distinct. 5

$x^3+3x^2-3=0$

3. (a) State Sandwich theorem. Prove that 1+3=4

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\,\ldots\ldots\ldots+\frac{1}{\sqrt{n^2+n}}\right)=1$$

(b) Define Cauchy sequence. Prove that a convergent sequence is a Cauchy sequence. 1+3=4

(c) Prove that $\lim\limits_{n\to\infty}(n)^{1/n}=1$ 3

(d) Prove that $\{2^n\}$ is a monotone increasing sequence. 3

Or

4. (a) Test the convergence of the series : 4

$$1+\frac{3}{\lfloor 2}+\frac{5}{\lfloor 3}+\frac{7}{\lfloor 4}+\,\ldots..$$

(b) Examine the convergence of the series 4

$$x+\frac{x^2}{2}+\frac{x^3}{3}+\,\ldots\ldots\ldots..,\ x>0$$

(c) Test the convergence of the series 3

$$1+\frac{1}{\lfloor 1}+\frac{2^2}{\lfloor 2}+\frac{3^2}{\lfloor 3}+\,\ldots\ldots\ldots$$

(d) Prove that an absolutely convergent series is convergent. 3

5. (a) State and prove De Moivre's theorem. 5

(b) Expand $\operatorname{Sin}^7\theta$ in a series of Sines of multiples of θ. 5

(c) If $\cos\alpha+\cos\beta+\cos\gamma=0$ and $\operatorname{Sin}\alpha+\operatorname{Sin}\beta+\operatorname{Sin}\gamma=0$, prove that $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$. 4

Or

6. (a) Prove that $\cos5\theta=16\cos^5\theta-20\cos^3\theta+5\cos\theta$ 4

(b) Find the principal value of $(1+i)^i$. 4

(c) Find the general solution of Sinz= 2. 4

(d) Find $\operatorname{Sin}^{-1}(2)$. 2

$\mathbb{R}$

7. (a) A relation ρ is defined on the set z by "ab if and only if a–b is divisible by 7" for a, b $\in$ z. Prove that ρ is an equivalence relation on z. 4

(b) Show that the mapping

f : $\quad\to\quad$ defined by f(x)= 2x+3, x $\in$ $\quad$ is bijective. 4

(c) Prove that in a group (G, 0) each element has only one inverse. 3

(d) Prove that ($\quad$, .) is not a group. 3

Or

8. (a) If each element, except the identity, of a group be or order 2, prove that the group is commutative. 3