show that both the partial derivatives exist at (0,0) but the function is not continuous thereat.

(b) If $u = \sin(x^2 + y^2)$, where $a^2x^2 + b^2y^2 = c^2$, find $\frac{du}{dx}$.

(c) Provide some examples where functions of several variables are needed to describe real life phenomena. 5+5+4=14

UNIT-V

9. a) Verify the linear dependence of the vectors $v = (1+i, 2i), w = (1, 1+i)$ in C^2 over ***C*** and over ***R***.

(b) Using Cayley-Hamilton theorem, find

, if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(c) Define basis and dimension of a vector space. Does every vector space have a basis? Justify.

OR

10. (a) Let V be the vector space of all 2×2 real symmetric matrices. Find the dimension of V.

(b) Find the values of and for which the equations

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

have (i) no solution (ii) a unique solution (iii) infinite number of solutions.

(c) Define eigenvalue and eigenvector with examples of each. 5+5+4=14

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A^8

B. Tech Odd Semester Examination, February, 2023

Agricultural Engineering / Computer Science & Engineering / Electronics & Communication Engineering

(1st Semester)

Course No.: ASH-102

(Mathematics-I)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

Note:
1. Attempt 05 (Five) questions by taking one form each unit.
2. Begin each answer in a new page.
3. Answer parts of a question at a place.
4. Assume reasonable data wherever required.
5. The figures in the right margin indicate full marks for the question.

UNIT-I

1. (a) Is the area under the curve $y = \frac{\ln x}{x^2}$ from $x = 1$ to $x = \infty$ finite? If so, find the area.

(b) Define different types of improper integrals with one example of each.

(c) Evaluate the improper integral $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

OR

2. (a) Establish the following relation between the Beta and the Gamma function:

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

(b) Evaluate:

(i) $\frac{\Gamma(3)\Gamma(2.5)}{\Gamma(5.5)}$ (ii) $\frac{6\Gamma(\frac{8}{3})}{5\Gamma(\frac{2}{3})}$.

(c) Evaluate the improper integral $\int_0^2 \frac{x^2}{\sqrt{(2-x)}}dx$.

5+5+4=14

UNIT-II

3. (a) Evaluate

(i) $\lim_{x\to\frac{\pi}{2}} \frac{\sin(x\cos x)}{\cos(x\sin x)}$

(ii) $\lim_{x\to\infty} \frac{x^n}{e^{kx}}, k$ is a constant.

(b) If $\frac{a}{3}+\frac{b}{2}+c=0$, then show that there exists a root of the equation $ax^2+bx+c=0$, which lies in (0, 1).

(c) Find the points at which the function $f(x)=(x-2)^3(x-3)^2$ has maxima and minima.

OR

4. (a) Find the extreme values (if any) of $x-\sin x$.

(b) Prove that $2x^3-3x^2-x+1=0$ has at least one root between 1 and 2.

(c) Prove that the largest rectangle with a given perimeter is a square. 5+5+4=14

UNIT-III

5. (a) Define a sequence and an infinite series with one example of each. Justify whether or not the examples of sequence and series given by you are convergent.

(b) Define a Cauchy sequence. Show that the sequence $\{(-3)^n\}_{n=0}^{\infty}$ is not Cauchy.

(c) Prove that $\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n})=0$.

OR

6. (a) Test the convergence of the following

(i) $\sum_{n=1}^{\infty}\frac{1}{n^2-n+1}$

(ii) $\sum_{n=1}^{\infty}\frac{1}{2^n-1}$

(b) Evaluate:

(i) $\sum_{n=1}^{\infty}\frac{1}{9}(\frac{1}{3})^{n-1}$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^n 5}{4^n}$.

(c) Discuss the convergence of $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$.

5+5+4=14

UNIT-IV

7. (a) Let $f(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2}, & x^4+y^2\neq 0\\ 0, & x^4+y^2=0.\end{cases}$

Evaluate $\lim_{(x,y)\to(0,0)} f(x,y)$.

b) Show that $\lim_{(x,y)\to(0,0)}\frac{2x^2y}{x^2+y^4}$ *does not exist.*

(c) Define partial derivative and total derivative with examples.

OR

8. (a) If $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$,