B. Tech Odd Semester Examination, February, 2023

Agricultural Engineering / Computer Science & Engineering / Electronics & Communication Engineering

(1st Semester)

Course No.: ASH-102 (Arrear)

(Mathematics-I)

Full Marks: 50
Pass Marks: 25

Time: 2 hours

Note:
1. Attempt any five questions.
2. Begin each answer in a new page.
3. Answer parts of a question at a place.
4. Assume reasonable data wherever required.
5. The figures in the right margin indicate full marks for the question.
6. All the mathematical symbols and abbreviations have their usual meanings.

1. a) Prove that the Euclidean 3-dimensional space is a vector space with respect to usual addition of vectors.

   b) Define basis and dimension of a vector space. Does every vector space has a basis? 5+5=10

2. a) Explain with examples the concepts of linear dependence and independence of vectors in avector space.

   b) If $u = \sin(x^2 + y^2)$,where $a^2x^2 + b^2y^2 = c^2$, find $\frac{du}{dx}$. 5+5=10

3. a) Prove that if any set of vectors contains the zero vector, then that set is linearly dependent.

b) Define a vector space, its basis and dimension. Give examples. 5+5=10

4. a) What is the diagonalization of a matrix? Write down some criteria that determine the diagonalizability of a matrix.

b) Is it possible to diagonalize the matrix $A=\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$? If yes, diagonalize it.

5+5=10

5. Consider the complex field C which contains the real field$R$ which contains the rational field Q.

(a) Show that C is a vector space over R and find its dimension.

(b) Show that R is a vector space over Q and find its dimension. 5+5=10

6. Find the characteristic equation of the matrix $A=\begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$. Hence find A^{-1}. 5+5=10

7. a) Verify if the vectors $v=(1+i,2i), w=(1,1+i)$ in C^2 are linearly dependent over C and over R.

b) Find the extreme values (if any) of $x-\sin x$.

5+5=10

8 a) Find the points at which the function $f(x)=(x-2)^3(x-3)^2$ has maxima and minima.

b) If , $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

show that both the partial derivatives exist at (0,0) but the function is not continuous thereat. 5+5=10

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