M. Tech Odd Semester Examination, February, 2023

Computer Science & Engineering

(1st Semester)

Course No.: MCSE-101

(Mathematical Foundation of Computer Science)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

Note:
1. Attempt 05 (Five) questions by taking one form each unit.
2. Begin each answer in a new page.
3. Answer parts of a question at a place.
4. Assume reasonable data wherever required.
5. The figures in the right margin indicate full marks for the question.

UNIT-I

1. (a) Let A be an arbitrary subset of some universe U then, show that of the following:

   (i) $A \cup A' = U$ (ii) $A \cup A' = \Phi$ 3+3=6

   (b) Prove that if S= {a, b, c} and T= {a, b, c, d} then proper subset of S is 2^n-1. 4

   (c) Prove that Φ be an empty set and A an arbitrary set, then $\Phi \subset A$. 4

2. (a) Is the 'divides' relation on the set of positive integers symmetric? Is it antisymmetric? 3+2=5

   (b) "<" relation on non-negative integer $a < b$, where a and b are non-negative integers.

Which of the relations are exists and provide proper explanation. 5

(c) The relation "<" defined on the set of positive integer. Is it partial order relation or not? 4

3. (a) Prove that the identity element of a subgroup is the same as that of group. 5

(b) Let $y = x^3 + 2x^2$, which properties holds on function? Explain with an example. 3+2=5

(c) Prove that a permutation on A is bijective function on A. 4

UNIT-II

4. (a) Show that every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion. 4

(b) If n is a positive integer and r is an integer with $1 < r < n$, then there are $P(n, r) = n(n-1)(n-2)\ldots\ldots(n-r+1)$. 5

(c) How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest? 5

5. (a) Let n and r be nonnegative integers with r dŠ n, then prove that C(n,r) = C(n, n-r). 4

(b) A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission (assuming that all crew members have the same job) 5

has an even number edges in common with any cut-set. 2+2+4=8

(b) Prove that the complete graph of five vertices is nonplanar. 6

UNIT-V

13. (a) Define a finite automata (FA) that accepts strings containing exactly 1 over alphabet {0, 1}. 7

(b) Convert the following regular grammar into regular expression:

$$S = 0B \mid 1A \mid \epsilon$$
$$A = 0S$$
$$B = 1S$$ 7

14. (a) Consider the CFG:

$$S \to aX$$
$$X \to aX \mid bX \mid \epsilon$$

What is the language this CFG generates? 7

(b) Write a CFG, which generates strings of balanced parenthesis. 7

15. (a) Draw a state diagram of an NFSA, which accepts binary strings with end '00' or '11'. 7

(b) Give an algorithm to simulate the behavior of an NFA. 7

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8. (a) An algorithm B solves problems of size n by recursively solving two subproblems of size n-1 and then combining the solutions in constant time. What is the complexity of this algorithm? 7

(b) Discuss the iterative binary search algorithm and analysis its time complexity. 7

9. (a) Write the algorithm to determine the subset of no overlapping intervals of maximum size, where input is considered as the set of intervals. Explain with an example. 4+3=7

(b) Why Dijkstra's algorithm is used of greedy approach? Write the Dijkstra's algorithm to find the single shortest path and analysis the time complexity. 2+3+2=7

UNIT-IV

10. (a) Prove that n vertices of complete graph is n(n-1)/2 edges. 7

(b) Show that a Hamiltonian circuit in a graph of n vertices consists of exactly n edges. 7

11. (a) What are the necessary conditions of isomorphic that two graphs are isomorphic? Prove that any two simple connected graphs with n vertices, all of degree two, are isomorphic. 3+4=7

(b) Prove that a Euler graph G is arbitrarily traceable from vertex V in G if and only if every circuit in G contains V. 7

12. (a) What do you mean by edge connectivity and vertex connectivity? Prove that every circuit

(c) How many bit strings of length n contain exactly r 1s? Explain with an example. 3+2=5

6. (a) What is the coefficient of $x^{12}y^{13}$in the expansion of $(2x-3y)^{25}$? 4

(b) Proof that the number of r-permutation of a set of n objects with repetition allowed is n. 5

(c) How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are nonnegative integer? 5

UNIT-III

7. (a) Find the upper bound for $f(n) = n^2 + 1$. 4

(b) Find the complexity of $T(n) = 4T(n/2) + n^2$ using master theoream for divide and conquer method. 5

(c) Write a recursive function for the running time T(n) of the function given below. Prove that using the iterative method that $T(n) = \Theta(n^3)$.

```
function (int n)
{
      if (n= =1)
         return;
       for (int i = 1; i <=n; i++)
         for (int j=1; j<=n; j++)
              printf (“*”);
                function (n-3);
}
```

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