UG (CBCS) EVEN SEMESTER EXAMINATION, 2023

EDUCATION (B.Sc. B.Ed.)

MATHEMATICS (PASS)

4th Semester

Course No. : BSMP - 402

(Differential Calculus, Integral Calculus and Differential Equation)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Prove that $\lim_{x\to 0}(1+3x)^{\frac{x+2}{x}} = e^6$ 3

(b) Prove that $\lim_{x\to 0}\frac{\log(1+x^3)}{Sin^3 x} = 1$ 3

(c) If $f(x) = \begin{cases} 3x+b & for \quad x>1 \\ 11 & for \quad x=1 \\ 5ax+b & for \quad x<1 \end{cases}$ 4

is continuous at $x = 1$, then find the value of a and b.

(d) Show that the function 4

$$f(x) = \begin{cases} 3+2x\ , & -\frac{3}{2} \le x \le 0 \\ 3-2x\ , & 0 < x \le \frac{3}{2} \end{cases}$$

is continuous at x = 0 but not differentiable at x = 0.

Or

2. (a) State Roll's theorem. Show that the equation $e^{x-1} + x - 2 = 0$ has no real root greater than 1. 4

(b) State Lagranges mean value theorem. Verify Lagranges mean value theorem for the function $f(x) = x(6 - x)$, where $3 \leq x \leq 4$ 1+3=4

(c) Find 'c' of the Lagranges mean value theorem, if $f(x) = x^2 - 3x - 1$, $x \in \left[\frac{-11}{7}, \frac{13}{7}\right]$ 3

(d) Examine the validity and conclusion of Roll's theorem for the function $f(x) = e^x \sin x$, $\forall\ x \in [0, \pi]$ 3

3. (a) Expand *Sin x* in an infinite series in powers of *x*. 4

(b) Expand *Cot x* in an infinite series in powers of *x*. 5

(c) Find the Maclaurin's series for *Coshx*. 5

Or

4. (a) Find the equation of tangent and normal to the curve $x = \frac{2at^2}{1+t^2}$, $y = \frac{2at^3}{1+t^2}$ 5

9. (a) Solve $\frac{d^2y}{dx^2} + y = x^2$ 5

(b) Solve $\frac{d^2y}{dx^2} + a^2 y = Sin\ ax$ 5

(c) Find the orthogonal trajectories to the family of curves $y^2 = 4ax$. 4

Or

10. Solve the following differential equation :

(a) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = Sin\ 2x$ 4

(b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x^2e^x$ 5

(c) $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 3y = e^{-x}$ 5

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(b) Prove that for the curve r = a (1 - cos), polar subtangent = $2a\,Sin^2\,\frac{\theta}{2}\tan\frac{\theta}{2}$ 5

(c) Find the radius of curvature at any point of the cycloid $x = a\,(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$

5. (a) Find the reduction formula for $\int Sin^n x\,dx$ where n is a positive integer. Evaluate $\int_0^{\frac{\pi}{2}} Sin^8 x\,dx$ 4+1=5

(b) Find the reduction formula for $\int_0^{\frac{\pi}{2}} Sin^m x\,.Cos^n x\,dx$ 5

(c) Evaluate $\int_0^{\frac{\pi}{2}} Sin^7 x\,.Cos^4 x\,dx$ 4

Or

6. (a) Find the surface area of the paraboloid of revolution of the parabola $y^2 = 4ax$, about x-axis and bounded by the section $x = x_1$. 4

(b) What is the entire area enclosed by the curve y = x^2 + 1 and the x-axis, between $x = 0$ and $x = 2$ 2

(c) Find the area enclosed by the curve $y = x^2 - 4x + 3$ and the x-axis, between $x = 1$ and $x = 4$. 3

(d) What is the entire area enclosed by the curve r = a cos 2θ 4

7. (a) Find the order, degree and linearity of the differential equation $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + x\,siny = 0$ 1+1+2=4

(b) Solve (cos y + y cos x) dx + (sin x - x sin y) dy = 0 3

(c) Solve $\frac{dy}{dx} + e^x y = e^x$ 3

(d) Solve $\frac{dy}{dx} + \frac{1}{x}y = x^2$ 4

Or

8. (a) Solve $Sec^2 y\,\frac{dy}{dx} + 2x\tan\,y = x^2$ 5

(b) Solve $x\,\frac{dy}{dx} + y = y^2\,logx$ 5

(c) Find the general solution and singular solution of the differential equation

$y = px + \frac{a}{p}$, $p = \frac{dy}{dx}$ 4