UG (CBCS) EVEN SEMESTER EXAMINATION, 2023

EDUCATION (B.Sc. B.Ed.)

MATHEMATICS (PASS)

6th Semester

Course No. : BSMP - 603

(Vectors, Statics and Dynamics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) If $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are three vectors such that $\bar{\alpha} + \bar{\beta} + \bar{\gamma} = \bar{0}$, and $|\bar{\alpha}| = 3$, $|\bar{\beta}| = 5$, $|\bar{\gamma}| = 7$, find the angle between $\bar{\alpha}$ and $\bar{\beta}$. 4

(b) If $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are three vectors such that $\bar{\alpha} + \bar{\beta} + \bar{\gamma} = \bar{0}$, prove that $\bar{\alpha} \times \bar{\beta} = \bar{\beta} \times \bar{\gamma} = \bar{\gamma} \times \bar{\alpha}$ 3

(c) If $\bar{\alpha} \times \bar{\beta} + \bar{\beta} \times \bar{\gamma} + \bar{\gamma} \times \bar{\alpha} = \bar{0}$, then show that $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ are coplanar. 3

(d) If $\bar{r} = a\cos t\, \hat{i} + a \sin t\, \hat{j} + a \tan \alpha\, \hat{k}$ then show that 4

$$\left| \frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} \right| = a^2 \sec \alpha$$

Or

2. (a) Find the directional derivative of $f = xy+yz+zx$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point (1, 2, 0) 4

   (b) show that $curl\ \bar{u} = \bar{o}$, if 3
   $\bar{u} = (y^2 + z^3)\,\hat{i} + (2xy - 5z)\,\hat{j} + (3xz^2 - 5y)\,\hat{k}$

   (c) shwo that the vector
   $\bar{v} = (x + 3y)\,\hat{i} + (y + az)\,\hat{j} + (x + az)\,\hat{k}$
   is solenoidal, if $a = -2$ 3

   (d) If $\bar{A} = x^2y\,\hat{i} - xz\,\hat{j} + 4yz\,\hat{k}$ then show that div curl $\bar{A} = 0$ and curl curl $\bar{A} = (4 + 2x)\,\hat{j}$ 4

3. (a) Three pieces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$, $y = 2$. Find the equation of the line of action of their resultant. 7

   (b) A particle is projected with a velocity u at an angle α to the horizon in a medium whose resistance is mkx (velocity). Find the motion of the particle. 7

Or

4. (a) Four rods, freely joined at their extremities A, B, C, D form a parallelogram. The opposite corners are joined by strins along the two diagonals, each of which is tight. Show that there tensions are proportional to the diagonals, along which the act.

10. (a) Find the radial and cross-redial accleration of a particle moving in a plane. 7

    (b) A straight smooth tube turns about one extremity 0 in a horizontal plane with uniform angular velocity w. Originally a particle is place in the tube at a distance a from 0 and projected towards 0 with a velocy v. Show that if $w < \frac{v}{a}$, the particle will reach 0 in time $\frac{1}{w} \tanh^{-1} \frac{aw}{v}$ 7

★★★

(b) A hermispherical shell rests on a rough inclined plane whose angle of friction is λ; show that the inclination of the plane base of the rim to the horizon cannot be greater than $\sin^{-1}(2\sin\lambda)$. 7

5. (a) Find the centroid of the arc of the parabola $y^2 = 4ax$ included between the vertex and an end point of the latus rectum. 7

(b) Find the centroid of the volumes formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about axis. 7

Or

6. (a) Find the law of force to the pole when the path is the cardioid $r = a(1 - \cos\theta)$, and prove that if F be the force at the apse and $\therefore$ v the velocity, $3v^2 = 4aF$. 7

(b) If a particle describes an ellipse under a force to its centre, show that the force is directly proportional to the distance from the centre. 7

7. (a) A particle is acted on 6y a force parallel to the axis of y where acceleration is ky and is initially projected with a velocity $a\sqrt{k}$ parallel to the axis of x at a point where y = a; prove that it will describe a catenary. 7

(b) A particle is describing a circle of radius a in such a way that its tangential acceleration is k times the normal acceleration where k is a

constant. If the speed of the particle at any point be u, prove that it will return to the same point after a time $\frac{a}{ku}(1 - e^{-2\pi K})$ 7

Or

8. (a) A particle project vertically upwards with a velocity u is met with air resistance which varies as the square of the velocity. Find the greatest height attained by the particle. 7

(b) If a body mass m executing simple harmonic motion makes n complete oscillations per sce, show that the difference of its K.E. when at its centre and when at a distance x from the centre, is given by $2m\pi^2n^2x^2$. 7

9. (a) An engine working at a constant rate H draws a load of mass M against a resistance R. Show that maximum speed attained s H/R and time taken attain half this speed is $\frac{MH}{R^2}\left(\log 2 - \frac{1}{2}\right)$ 7

(b) A shell of mass m is ejected from a gun of mass M by an explosion which generates kinetic energy E. Prove that the initial velocity of the shell is $\sqrt{\frac{2\,ME}{(M+m)m}}$ 7

Or