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Elastic Waves in a Homogeneous/ an Inhomogeneous Fibre-Reinforced Elastic Half-Spaces

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Abstract—The problem of reflection and refraction of elastic waves due to incident quasi P and quasi SV-waves at a plane interface between the homogeneous and inhomogeneous fibre-reinforced elastic half-spaces has been investigated. The reflection and refraction coefficients and energy ratios of the reflected and refracted quasi P and quasi SV-waves are obtained. These coefficients and ratios are computed numerically for a particular model and the results are shown graphically.

Keywords: Fibre-reinforced Elastic Half-space, *qP* and *qSV*-waves, Reflection and Refraction Coefficients, Energy Ratio

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INTRODUCTION

Spencer (1972) gave the concept of deformation in fibre-reinforced elastic materials. The concrete and steel components of fibre-reinforced materials act together as a single anisotropic unit as long as they remain in the elastic condition. Such composite materials are used in many engineering structures due to high strength and lightweight. The stress in elastic plates is reinforced by fibers lying in concentric circles (Belfield *et al*. 1983) with anisotropic characters. The reflection of quasi P and quasi SV-waves at a plane free and rigid boundaries of a fibre-reinforced elastic medium were investigated (Chattopadhyay *et al.* 2002) and the authors obtained the phase velocity of quasi P and quasi SV-waves. Singh and Singh (2004) discussed the reflection of plane waves at the free surface of a fibre-reinforced elastic half-space and derived the closed form expressions of the amplitude ratios for reflected qSV and qP-waves. The scattering of elastic waves from a corrugated interface between two dissimilar fibre-reinforced elastic half-spaces were discussed by Singh and Tomar (2007a) and they obtained the reflection and transmission coefficients with the help of the Rayleigh method of approximation. The propagation of plane waves and frequency equations (Othman *et al.* 2012) were discussed for a thermally conducting linear fibre-reinforced composite materials. Abbas (2011) studied the two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation and compared the results with without energy dissipation. The amplitude and energy ratios of the reflected waves [18] due to incident longitudinal wave are obtained for the plane free boundary in a fibre-reinforced thermoelastic half-space.

The subject of elastic wave propagation is very helpful in determining the characteristics of the material body. It is very common in the field of earthquake engineering, Geophysics, Seismology, etc. There are many open literatures related with the problems of wave propagation in fibre-reinforced composite materials and notable among them are Abbas and Abd-alla (2011); Bedford and Sutherland (1973); Cates and Edwards (1984), Chattopadhyay and Co-workers (2007, 2012); Chen *et al.* (2011); Green (1991); Othman *et al.* (2012); Singh (2007); Singh and Tomar (2007b), Zorammuana and Singh (2015a, b) and Tomar and Singh (2006).

Keeping the importance of elastic wave propagation, we have attempted the problem of reflection and refraction of elastic waves due to incident qP and qSV-waves at a plane interface between the homogeneous/inhomogeneous fibre-reinforced elastic half-spaces. Two directional fibre-orientation is used and this makes the problem more

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interesting. The reflection and refraction coefficients with energy ratios of the reflected and refracted qP and qSV-waves are obtained. These coefficients are computed with the help of MatLab for a particular model.

BASIC EQUATIONS

The constitutive relations for a linearly fibre-reinforced elastic medium are given by Belfield *et al*. (1983) as

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j, \quad (i,j,k,m=1,2,3) \tag{1}$$

where τ_{ij} are stress tensors and e_{ij} are strain tensors with $e_{ij} = \frac{1}{2}(u_{i,j}+u_{j,i}), \lambda$, μ_T and μ_L are elastic constants, α and β are specific stress components depending upon the concrete part of the composite materials, δ_{ij} is Kronecker delta and α={$(a_1, a_2, a_3); a_1^2 + a_2^2 + a_3^2 = 1$}.

The equation of motion in a fibre-reinforced elastic material in the absence of body force is given by

$$\tau_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \qquad (i,j=1,2,3) \tag{2}$$

where $u_i = (u_1, u_2, u_3)$ and $(x_1, x_2, x_3) \equiv (x, y, z)$.

We take two dimensional wave propagation in *xz*-plane with the preferred direction ofibre-reinforcement as $(a_1, 0, a_3)$. Eq. (2) may be re-written as

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{13}}{\partial z} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{3}$$

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{4}$$

$$\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{5}$$

where $\tau_{11} = \{\lambda + 2\mu_T + 2\alpha a_1^2 + 4(\mu_L - \mu_T)a_1^2 + \beta a_1^4\}\frac{\partial u_1}{\partial x} + (\lambda + \alpha + \beta a_1^2 a_3^2)\frac{\partial u_3}{\partial z} + \{\alpha a_1 a_3 + 2(\mu_L - \mu_T)a_1 a_3 + \beta a_1^3 a_3\}\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right),$

$$\tau_{13} = \{\alpha a_1 a_3 + 2(\mu_L - \mu_T)a_1 a_3 + \beta a_1^3 a_3\}\frac{\partial u_1}{\partial x} + (\mu_L + \beta a_1^2 a_3^2)\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right) + \{\alpha a_1 a_3 + 2(\mu_L - \mu_T)a_1 a_3 + \beta a_1 a_3^3\}\frac{\partial u_3}{\partial z},$$

$$\tau_{21} = \mu_T \frac{\partial u_2}{\partial x} + (\mu_L - \mu_T)a_1\left(a_1\frac{\partial u_2}{\partial x} + a_3\frac{\partial u_2}{\partial z}\right),$$

$$\tau_{23} = \mu_T \frac{\partial u_2}{\partial z} + (\mu_L - \mu_T)a_3\left(a_1\frac{\partial u_2}{\partial x} + a_3\frac{\partial u_2}{\partial z}\right),$$

$$\tau_{33} = (\lambda + \alpha + \beta a_1^2 a_3^2)\frac{\partial u_1}{\partial x} + \{\lambda + 2\mu_T + 2\alpha a_3^2 + 4(\mu_L - \mu_T)a_3^2 + \beta a_3^4\}\frac{\partial u_3}{\partial z} + \{\alpha a_1 a_3 + 2(\mu_L - \mu_T)a_1 a_3 + \beta a_1 a_3^3\}\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right).$$

These are the equations of motion for the homogeneous and inhomogeneous fibre-reinforced elastic materials without body force.

PROBLEM FORMULATION

Let x and y-axes of the Cartesian coordinate system be on the horizontal plane and z-axis be pointing vertically downward. Consider a homogeneous fibre-reinforced elastic half-space $\{H : z \geq 0\}$ with $\lambda, \alpha, \beta, \mu_L, \mu_T$ as elastic constants and as density; and an inhomogeneous fibre-reinforced elastic half-space $\{H' : z \leq 0\}$ with $\lambda', \alpha', \beta', \mu_L', \mu_T'$ as elastic constants and ρ' as density separated by $z = 0$. The elastic constants and density in the inhomogeneous fibre-reinforced elastic half-space, H' may be defined as [Chattopadhyay and Singh, 2012]

$$\mu_L' = \mu_L^{(0)}(1 - \varepsilon \cos sz), \quad \mu_T' = \mu_T^{(0)}(1 - \varepsilon \cos sz),$$
$$\rho' = \rho_0(1 - \varepsilon \cos sz),$$
$$\alpha' = \alpha^{(0)}(1 - \varepsilon \cos sz), \quad \beta' = \beta^{(0)}(1 - \varepsilon \cos sz),$$
$$\lambda' = \lambda^{(0)}(1 - \varepsilon \cos sz),$$

where ε is small positive constant and s is real depth parameter.

The equations of motion for *qP* and *qSV*-waves in the homogeneous fibre-reinforced elastic medium, H are

$$R_1\frac{\partial^2 u_1}{\partial x^2}+R_2\frac{\partial^2 u_1}{\partial x\partial z}+R_3\frac{\partial^2 u_1}{\partial z^2}+R_4\frac{\partial^2 u_3}{\partial x^2}+R_5\frac{\partial^2 u_3}{\partial x\partial z}+R_6\frac{\partial^2 u_3}{\partial z^2}=\rho\frac{\partial^2 u_1}{\partial t^2}, \quad (6)$$

$$R_4\frac{\partial^2 u_1}{\partial x^2}+R_5\frac{\partial^2 u_1}{\partial x\partial z}+R_6\frac{\partial^2 u_1}{\partial z^2}+R_3\frac{\partial^2 u_3}{\partial x^2}+R_7\frac{\partial^2 u_3}{\partial x\partial z}+R_8\frac{\partial^2 u_3}{\partial z^2}=\rho\frac{\partial^2 u_3}{\partial t^2}, \quad (7)$$

where

$$R_1=\lambda+2\mu_T+2\alpha a_1^2+4(\mu_L-\mu_T)a_1^2+\beta a_1^4,$$
$$R_2=2[\alpha a_1a_3+2(\mu_L-\mu_T)a_1a_3+\beta a_1^3a_3],$$
$$R_3=\mu_L+\beta a_1^2a_3^2,$$
$$R_4=\alpha a_1a_3+2(\mu_L-\mu_T)a_1a_3+\beta a_1^3a_3,$$
$$R_5=\lambda+\alpha+\mu_L+2\beta a_1^2a_3^2,$$
$$R_6=\alpha a_1a_3+2(\mu_L-\mu_T)a_1a_3+\beta a_1a_3^3,$$
$$R_7=2[\alpha a_1a_3+2(\mu_L-\mu_T)a_1a_3+\beta a_1a_3^3],$$
$$R_8=\lambda+2\mu_T+2\alpha a_3^2+4(\mu_L-\mu_T)a_3^2+\beta a_3^4.$$

Similarly, the equations of motion for *qP* and *qSV*-waves in the inhomogeneous fibre-reinforced elastic medium, H' are given by

$$R_1^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial x^2}+R_2^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial x\partial z}+R_3^{(0)}\left\{(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial z^2}+\varepsilon s\sin sz\left(\frac{\partial u_1'}{\partial z}+\frac{\partial u_3'}{\partial x}\right)\right\}+R_4^{(0)}\{(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial x^2}+\varepsilon\sin sz\frac{\partial u_1'}{\partial x}\}+R_5^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial x\partial z}+R_6^{(0)}\{(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial z^2}+\varepsilon s\sin sz\frac{\partial u_3'}{\partial z}\}=\rho_0(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial t^2}, \quad (8)$$

$$R_4^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial x^2}+R_5^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial x\partial z}+R_6^{(0)}\{(1-\varepsilon\cos sz)\frac{\partial^2 u_1'}{\partial z^2}+\varepsilon s\sin sz(\frac{\partial u_1'}{\partial z}+\frac{\partial u_3'}{\partial x})\}+R_3^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial x^2}+R_7^{(0)}(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial x\partial z}+R_8^{(0)}\{(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial z^2}+\varepsilon s\sin sz\frac{\partial u_3'}{\partial z}\}+R_9^{(0)}\varepsilon s\sin sz\frac{\partial u_1'}{\partial x}=\rho_0(1-\varepsilon\cos sz)\frac{\partial^2 u_3'}{\partial t^2}, \quad (9)$$

where

$$R_1^{(0)}=\lambda^{(0)}+2\mu_T^{(0)}+2\alpha^{(0)}a_1^2+4(\mu_L^{(0)}-\mu_T^{(0)})a_1^2+\beta^{(0)}a_1^4,$$
$$R_2^{(0)}=2\{\alpha^{(0)}a_1a_3+2(\mu_L^{(0)}-\mu_T^{(0)})a_1a_3+\beta^{(0)}a_1^3a_3\},$$
$$R_3^{(0)}=\mu_L^{(0)}+\beta^{(0)}a_1^2a_3^2,$$
$$R_4^{(0)}=\alpha^{(0)}a_1a_3+2(\mu_L^{(0)}-\mu_T^{(0)})a_1a_3+\beta^{(0)}a_1^3a_3,$$
$$R_5^{(0)}=\lambda^{(0)}+\alpha^{(0)}+\mu_L^{(0)}+2\beta^{(0)}a_1^2a_3^2,$$
$$R_6^{(0)}=\alpha^{(0)}a_1a_3+2(\mu_L^{(0)}-\mu_T^{(0)})a_1a_3+\beta^{(0)}a_1a_3^3,$$
$$R_7^{(0)}=2\{\alpha^{(0)}a_1a_3+2(\mu_L^{(0)}-\mu_T^{(0)})a_1a_3+\beta^{(0)}a_1a_3^3\},$$
$$R_8^{(0)}=\lambda^{(0)}+2\mu_T^{(0)}+2\alpha^{(0)}a_3^2+4(\mu_L^{(0)}-\mu_T^{(0)})a_3^2+\beta^{(0)}a_3^4,$$
$$R_9^{(0)}=\lambda^{(0)}+\alpha^{(0)}+\beta^{(0)}a_1^2a_3^2.$$

WAVE PROPAGATION

Suppose a plane wave (*qP* or *qSV*-wave) with amplitude, A_0 making an angle θ_0 with the normal be incident at the plane interface, $z=0$ between homogeneous and inhomogeneous fibre-reinforced elastic half-spaces. The incident plane wave give rises reflected *qP* and *qSV*-waves in the half-space, H and refracted *qP* and *qSV*-waves in the half-space, H'.

The total displacement components due to incident and reflected waves in the half space, H is given by

$$u_1=\sum_{j=0}^{2}A_j\exp[ik_j\{c_jt-(x\sin\theta_j\pm z\cos\theta_j)\}], \quad (10)$$

$$u_3=\sum_{j=0}^{2}\eta_jA_j\exp[ik_j\{c_jt-(x\sin\theta_j\pm z\cos\theta_j)\}], \quad (11)$$

where '+' sign corresponds for the incident plane wave and '−' sign corresponds for the reflected waves, $\omega(=k_jc_j)$ is the angular frequency, A_j are the amplitude constants at angles θ_j and expressions of the coupling constants η_0, and η_j are given as

$$\eta_0 = \frac{-(R_4\sin^2\theta_0 + R_5\sin\theta_0\cos\theta_0 + R_6\cos^2\theta_0)}{R_3\sin^2\theta_0 + R_7\sin\theta_0\cos\theta_0 + R_8\cos^2\theta_0 - \rho c_0^2},$$

$$\eta_j = \frac{-(R_4\sin^2\theta_j - R_5\sin\theta_j\cos\theta_j + R_6\cos^2\theta_j)}{R_3\sin^2\theta_j - R_7\sin\theta_j\cos\theta_j + R_8\cos^2\theta_j - \rho c_j^2}.$$

Similarly, the displacement components in the half space, H' are given by

$$u_1' = \sum_{j=3}^{4} A_j \exp[ik_j\{c_j t - (x\sin\theta_j + z\cos\theta_j)\}], \quad (12)$$

$$u_3' = \sum_{j=3}^{4} \eta_j A_j \exp[ik_j\{c_j t - (x\sin\theta_j + z\cos\theta_j)\}], \quad (13)$$

where A_j are the amplitude constants at angles θ_j and expressions of the coupling constants, η_j may be written as

$$\eta_j = \frac{-(R_4^{(0)}\sin^2\theta_j + R_5^{(0)}\sin\theta_j\cos\theta_j + R_6^{(0)}\cos^2\theta_j)}{R_3^{(0)}\sin^2\theta_j + R_7^{(0)}\sin\theta_j\cos\theta_j + R_8^{(0)}\cos^2\theta_j - \rho c_j^2}.$$

The phase velocities of the reflected and refracted waves are given by

$$\rho c_{1,2}^2 = \frac{U \pm \sqrt{U^2 - 4V}}{2}, \quad (14)$$

$$\rho c_{3,4}^2 = \frac{U^{(0)} \pm \sqrt{U^{(0)^2} - 4V^{(0)}}}{2}, \quad (15)$$

where $'+'$ sign corresponds for the *qP* wave, while $'-'$ sign corresponds for the *qSV* wave and the expressions of $U, V, U^{(0)}$ and $V^{(0)}$ are given by

$$U = R_1p_1^2 + R_2p_1p_3 + R_3 + R_7p_1p_3 + R_8p_3^2,$$

$$\begin{aligned}V = {} & R_1R_3p_1^4 + R_1R_7p_1^3p_3 + R_1R_8p_1^2p_3^2 + R_2R_3p_1^3p_3 \\ & + R_2R_7p_1^2p_3^2 + R_2R_8p_1p_3^3 + R_3^2p_1^2p_3^2 \\ & + R_3R_7p_3^3p_1 + R_3R_8p_3^4 - (R_4^2p_1^4 + 2R_4R_5p_1^3p_3 \\ & + 2R_4R_6p_1^2p_3^2 + R_5^2p_1^2p_3^2 + 2R_5R_6p_1p_3^3 + R_6^2p_3^4),\end{aligned}$$

$$\begin{aligned}U^{(0)} = {} & R_1^{(0)}p_1^2 + R_2^{(0)}p_1p_3 + R_3^{(0)} + R_3^{(0)}L + R_4^{(0)}M \\ & + R_6^{(0)}M + R_7^{(0)}p_1p_3 + R_8^{(0)}(p_3^2 + L),\end{aligned}$$

$$\begin{aligned}V^{(0)} = {} & R_1^{(0)}R_6^{(0)}p_1^2M + R_1^{(0)}R_3^{(0)}p_1^4 + R_1^{(0)}R_7^{(0)}p_1^3p_3 \\ & + R_1^{(0)}R_8^{(0)}(p_1^2p_3^2 + p_1^2L) + R_2^{(0)}R_6^{(0)}p_1p_3M \\ & + R_2^{(0)}R_3^{(0)}p_1^3p_3 + R_2^{(0)}R_7^{(0)}p_1^2p_3^2 \\ & + R_2^{(0)}R_8^{(0)}(p_1p_3^3 + p_1p_3L) + R_3^{(0)^2}(p_1^2p_3^2 + p_1^2L) \\ & + R_3^{(0)}R_7^{(0)}(p_1p_3^3 + p_1p_3L) + R_3^{(0)}R_8^{(0)}(p_3^2 + L)^2 + R_4^{(0)}R_6^{(0)}M^2 \\ & + R_4^{(0)}R_7^{(0)}p_1p_3M + R_4^{(0)}R_8^{(0)}(p_3^2M + LM) - R_3^{(0)}R_5^{(0)}p_1p_3M \\ & - R_3^{(0)}R_9^{(0)}M^2 - R_4^{(0)^2}p_1^4 - 2R_4^{(0)}R_5^{(0)}p_1^3p_3 \\ & - 2R_4^{(0)}R_6^{(0)}(p_1^2p_3^2 + p_1^2L) - R_4^{(0)}R_9^{(0)}p_1^2M - R_5^{(0)^2}p_1^2p_3^2 \\ & - 2R_5^{(0)}R_6^{(0)}(p_1p_3^3 + p_1p_3L) - R_5^{(0)}R_9^{(0)}p_1p_3M \\ & - R_6^{(0)^2}(p_3^2 + L)^2 - R_6^{(0)}R_9^{(0)}(p_3^2M + LM),\end{aligned}$$

$$L = \frac{i\varepsilon p_3\sin sz}{(1-\varepsilon\cos sz)k}, \quad M = \frac{i\varepsilon p_1\sin sz}{(1-\varepsilon\cos sz)k}.$$

The Snell's law of this problem is given by

$$\frac{\sin\theta_0}{c_0} = \frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2} = \frac{\sin\theta_3}{c_3} = \frac{\sin\theta_4}{c_4}. \quad (14)$$

BOUNDARY CONDITIONS

The boundary conditions are the continuity of the displacement components and stress tensors at the interface, . Mathematically, at $z = 0$

(i) continuity of displacements

$$u_1 = u_1', \quad u_3 = u_3', \quad (15)$$

(ii) continuity of the stress tensors

$$\tau_{33} = \tau_{33}', \quad \tau_{13} = \tau_{13}'. \quad (16)$$

The continuity of stress tensors may be written in terms of displacement components as

$$R_8\frac{\partial u_3}{\partial z} - q_1\frac{\partial u_1}{\partial x} + R_6\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right) = (1-\varepsilon)$$
$$\left[R_9^{(0)}\frac{\partial u_1'}{\partial x} + R_8^{(0)}\frac{\partial u_3'}{\partial z} + R_6^{(0)}\left(\frac{\partial u_1'}{\partial z} + \frac{\partial u_3'}{\partial x}\right)\right], \quad (17)$$

$$R_6\frac{\partial u_3}{\partial z} - m_1\frac{\partial u_1}{\partial x} + R_3\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}\right) = (1-\varepsilon)$$
$$\left[R_4^{(0)}\frac{\partial u_1'}{\partial x} + R_3^{(0)}\left(\frac{\partial u_1'}{\partial z} + \frac{\partial u_3'}{\partial x}\right) + R_6^{(0)}\frac{\partial u_3'}{\partial z}\right]. \quad (18)$$

Using Eqs. (10)-(13) and (16) into the boundary conditions (17) and (18), we have

$$BZ 5 D, \tag{19}$$

where

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{pmatrix}, \quad Z = \begin{pmatrix} A_1/A_0 \\ A_2/A_0 \\ A_3/A_0 \\ A_4/A_0 \end{pmatrix},$$

$$D = \begin{pmatrix} -1 \\ -1 \\ -\Delta \\ -\Delta' \end{pmatrix},$$

and

$$b_{11} = \frac{\eta_1}{\eta_0}, b_{12} = \frac{\eta_2}{\eta_0}, b_{13} = -\frac{\eta_3}{\eta_0}, b_{14} = -\frac{\eta_4}{\eta_0},$$

$$b_{21} = q_1 k_0 \sin\theta_0 + R_8\eta_1 k_1 \cos\theta_1 + R_6 k_1 \cos\theta_1$$
$$-R_6\eta_1 k_0 \sin\theta_0,$$
$$b_{22} = q_1 k_0 \sin\theta_0 + R_8\eta_2 k_2 \cos\theta_2 + R_6 k_2 \cos\theta_2$$
$$-R_6\eta_2 k_0 \sin\theta_0,$$
$$b_{23} = (1-\varepsilon)(R_9^{(0)} k_0 \sin\theta_0 + R_8^{(0)}\eta_3 k_3 \cos\theta_3$$
$$+R_6^{(0)} k_3 \cos\theta_3 + R_6^{(0)}\eta_3 k_0 \sin\theta_0),$$
$$b_{24} = (1-\varepsilon)(R_9^{(0)} k_0 \sin\theta_0 + R_8^{(0)}\eta_4 k_4 \cos\theta_4$$
$$+R_6^{(0)} k_4 \cos\theta_4 + R_6^{(0)}\eta_4 k_0 \sin\theta_0),$$
$$b_{31} = m_1 k_0 \sin\theta_0 + R_3 k_1 \cos\theta_1 - R_3\eta_1 k_0 \sin\theta_0$$
$$+R_6\eta_1 k_1 \cos\theta_1,$$
$$b_{32} = m_1 k_0 \sin\theta_0 + R_3 k_2 \cos\theta_2 - R_3\eta_2 k_0 \sin\theta_0$$
$$+R_6\eta_2 k_2 \cos\theta_2,$$
$$b_{33} = (1-\varepsilon)(R_4^{(0)} k_0 \sin\theta_0 + R_3^{(0)} k_3 \cos\theta_3$$
$$+R_3^{(0)}\eta_3 k_0 \sin\theta_0 + R_6^{(0)}\eta_3 k_3 \cos\theta_3),$$
$$b_{34} = (1-\varepsilon)(R_4^{(0)} k_0 \sin\theta_0 + R_3^{(0)} k_4 \cos\theta_4$$
$$+R_3^{(0)}\eta_4 k_0 \sin\theta_0 + R_6^{(0)}\eta_4 k_4 \cos\theta_4),$$
$$\Delta = q_1 k_0 \sin\theta_0 - R_8\eta_0 k_0 \cos\theta_0 - R_6 k_0 \cos\theta_0$$
$$-R_6\eta_0 k_0 \sin\theta_0,$$
$$\Delta' = m_1 k_0 \sin\theta_0 - R_3 k_0 \cos\theta_0 - R_3\eta_0 k_0 \sin\theta_0$$
$$-R_6\eta_0 k_0 \cos\theta_0,$$
$$q_1 = -(\lambda + \alpha + \beta a_1^2 a_3^2), m_1 = -\{\alpha a_1 a_3 + 2(\mu_L$$
$$-\mu_T) a_1 a_3 + \beta a_1^3 a_3\}.$$

These equations will help to find the reflection and refraction coefficients of the reflected and refracted qP and qSV-waves.

SOLUTION OF THE PROBLEM

With the help of Eq. (19), we obtain the amplitude constants of the reflected and refracted waves as

$$\frac{A_1}{A_0} = \frac{D_1}{D_0}, \frac{A_2}{A_0} = \frac{D_2}{D_0}, \frac{A_3}{A_0} = \frac{D_3}{D_0}, \frac{A_4}{A_0} = \frac{D_4}{D_0}, \tag{20}$$

where

$$D_0 = \begin{vmatrix} 1 & 1 & -1 & -1 \\ b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{vmatrix},$$

$$D_1 = \begin{vmatrix} -1 & 1 & -1 & -1 \\ -1 & b_{12} & b_{13} & b_{14} \\ -\Delta & b_{22} & b_{23} & b_{24} \\ -\Delta' & b_{32} & b_{33} & b_{34} \end{vmatrix},$$

$$D_2 = \begin{vmatrix} 1 & -1 & -1 & -1 \\ b_{11} & -1 & b_{13} & b_{14} \\ b_{21} & -\Delta & b_{23} & b_{24} \\ b_{31} & -\Delta' & b_{33} & b_{34} \end{vmatrix},$$

$$D_3 = \begin{vmatrix} 1 & 1 & -1 & -1 \\ b_{11} & b_{12} & -1 & b_{14} \\ b_{21} & b_{22} & -\Delta & b_{24} \\ b_{31} & b_{32} & -\Delta' & b_{34} \end{vmatrix},$$

$$D_4 = \begin{vmatrix} 1 & 1 & -1 & -1 \\ b_{11} & b_{12} & b_{13} & -1 \\ b_{21} & b_{22} & b_{23} & -\Delta \\ b_{31} & b_{32} & b_{33} & -\Delta' \end{vmatrix}.$$

Using the amplitude constants corresponding to incident displacement components, u_1 and u_3, the amplitude of the incident wave is given by

$$\sqrt{A_0^2 + \eta_0^2 A_0^2} = \sqrt{1+\eta_0^2}A_0.$$

Similarly, the amplitude of the reflected and refracted waves are given by $\sqrt{1+\eta_j^2}A_j, (j = 1,2,3,4)$. Thus, reflection coefficients (r_{ip}, r_{is}) and refraction coefficients (t_{ip}, t_{is}) are given by

$$r_{ip} = \sqrt{\frac{1+\eta_1^2}{1+\eta_0^2}}\frac{A_1}{A_0}, \quad r_{is} = \sqrt{\frac{1+\eta_2^2}{1+\eta_0^2}}\frac{A_2}{A_0}, \tag{21}$$

$$t_{ip} = \sqrt{\frac{1+\eta_3^2}{1+\eta_0^2}}\frac{A_1}{A_0}, \quad t_{is} = \sqrt{\frac{1+\eta_4^2}{1+\eta_0^2}}\frac{A_2}{A_0}. \tag{22}$$

The energy transmission per unit area at the interface, $z = 0$ may be written as[3]

$$\wp^* = \tau_{31}\cdot\dot{u}_1 + \tau_{33}\cdot\dot{u}_3 + \tau'_{31}\cdot\dot{u}'_1 + \tau'_{33}\cdot\dot{u}'_3. \tag{23}$$

The expression of the energy due to incident wave is given by

$$E_{inc} = -k_0\omega J_0 A_0^2 \exp[2i\{\omega t - k_0(x\sin\theta_0 + z\cos\theta_0)\}], \tag{24}$$

where

$$J_0 = \{q_1\sin\theta_0 - R_8\eta_0\cos\theta_0 - R_6(\cos\theta_0 + \eta_0\sin\theta_0)\}\eta_0 + m_1\sin\theta_0 - R_3(\cos\theta_0 + \eta_0\sin\theta_0) - R_6\eta_0\cos\theta_0.$$

The modulus of energy ratios of the reflected and refracted *qP* and *qSV*-waves are given as

$$E_1 = \left|\frac{k_1 J_1}{k_0 J_0}\right|\left|\frac{A_1}{A_0}\right|^2, \quad E_2 = \left|\frac{k_2 J_2}{k_0 J_0}\right|\left|\frac{A_2}{A_0}\right|^2,$$

$$E_3 = \left|\frac{k_3 J_3}{k_0 J_0}\right|\left|\frac{A_3}{A_0}\right|^2, \quad E_4 = \left|\frac{k_4 J_4}{k_0 J_0}\right|\left|\frac{A_4}{A_0}\right|^2, \tag{25}$$

where the expressions of J_1, J_2, J_3 and J_4 are given by

$$J_1 = \{q_1\sin\theta_1 + R_8\eta_1\cos\theta_1 + R_6(\cos\theta_1 - \eta_1\sin\theta_1)\}\eta_1 + m_1\sin\theta_1 + R_3(\cos\theta_1 - \eta_1\sin\theta_1) + R_6\eta_1\cos\theta_1,$$

$$J_2 = \{q_1\sin\theta_2 + R_8\eta_2\cos\theta_2 + R_6(\cos\theta_2 - \eta_2\sin\theta_2)\}\eta_2 + m_1\sin\theta_2 + R_3(\cos\theta_2 - \eta_2\sin\theta_2) + R_6\eta_2\cos\theta_2,$$

$$J_3 = (1-\varepsilon)[\{R_9^{(0)}\sin\theta_3 + R_8^{(0)}\eta_3\cos\theta_3 + R_6^{(0)}(\cos\theta_3 + \eta_3\sin\theta_3)\}\eta_3 + R_4^{(0)}\sin\theta_3 + R_3^{(0)}(\cos\theta_3 + \eta_3\sin\theta_3) + R_6^{(0)}\eta_3\cos\theta_3],$$

$$J_4 = (1-\varepsilon)[\{R_9^{(0)}\sin\theta_4 + R_8^{(0)}\eta_4\cos\theta_4 + R_6^{(0)}(\cos\theta_4 + \eta_4\sin\theta_4)\}\eta_4 + R_4^{(0)}\sin\theta_4 + R_3^{(0)}(\cos\theta_4 + \eta_4\sin\theta_4) + R_6^{(0)}\eta_4\cos\theta_4].$$

We have seen that the energy ratios are functions of the amplitude ratios, elastic constants, fibre orientation, inhomogeneity parameter and the angle of incidence. We have observed that these coefficients depend on the elastic constants, fibre orientation, inhomogeneity parameter and the angle of incidence.

If *qP*-wave is incident, then $\theta = \theta$, c c and the reflection coefficients (r_{pp}, r_{ps}) and refraction coefficients (t_{pp}, t_{ps}) of the reflected and refracted *qP* and *qSV*-waves are given by Eqs. (21) and (22) with $i = p$. Moreover, if the incident wave is *qSV*-wave, then $\theta_0 = \theta_2$, $c_0 = c_2$ and the reflection coefficients (r_{sp}, r_{ss}) and refraction coefficients (t_{sp}, t_{ss}) of the reflected and refracted *qP* and *qSV*-waves are given by Eqs. (21) and (22) with $i = s$. Eq. (25) gives their energy ratios of the reflected and refracted *qP* and *qSV*-waves.

PARTICULAR CASES

Case I: If the half-space, H' reduces to homogeneous fibre-reinforced elastic medium, the problem reduces to the reflection and refraction of elastic waves at the plane interface between the two dissimilar homogeneous fibre-reinforced elastic half-spaces. With $\varepsilon = 0$ and the phase velocities of *qP* and *qSV*-waves in the half space, H' are given by Eq. (15) with the following modified values

$$U^{(0)} = R_1^{(0)}p_1^2 + R_2^{(0)}p_1p_3 + R_3^{(0)} + R_7^{(0)}p_1p_3 + R_8^{(0)}p_3^2,$$

$$V^{(0)} = R_1^{(0)}R_3^{(0)}p_1^4 + R_1^{(0)}R_7^{(0)}p_1^3p_3 + R_1^{(0)}R_8^{(0)}p_1^2p_3^2 + R_2^{(0)}R_3^{(0)}p_1^3p_3 + R_2^{(0)}R_7^{(0)}p_1^2p_3^2 + R_2^{(0)}R_8^{(0)}p_1p_3^3$$

$$+R_3^{(0)^2} p_1^2 p_3^2 + R_3^{(0)} R_7^{(0)} p_1 p_3^3 + R_3^{(0)} R_8^{(0)} p_3^4 - R_4^{(0)^2} p_1^4$$
$$-2R_4^{(0)} R_5^{(0)} p_1^3 p_3 - 2R_4^{(0)} R_6^{(0)} p_1^2 p_3^2 - R_5^{(0)^2} p_1^2 p_3^2$$
$$-2R_5^{(0)} R_6^{(0)} p_1 p_3^3 - R_6^{(0)^2} p_3^4.$$

The reflection and refraction coefficients and energy ratios corresponding to the reflected and refracted *qP* and *qSV*-waves are given by Eqs. (21)-(22) and (25) with the following changes

$$b_{23} = R_9^{(0)} k_3 \sin\theta_3 + R_8^{(0)} \eta_3 k_3 \cos\theta_3 + R_6^{(0)} k_3 \cos\theta_3$$
$$+R_6^{(0)} \eta_3 k_3 \sin\theta_3,$$
$$b_{24} = R_9^{(0)} k_4 \sin\theta_4 + R_8^{(0)} \eta_4 k_4 \cos\theta_4 + R_6^{(0)} k_4 \cos\theta_4$$
$$+R_6^{(0)} \eta_4 k_4 \sin\theta_4,$$
$$b_{33} = R_4^{(0)} k_3 \sin\theta_3 + R_3^{(0)} k_3 \cos\theta_3 + R_3^{(0)} \eta_3 k_3 \sin\theta_3$$
$$+R_6^{(0)} \eta_3 k_3 \cos\theta_3,$$
$$b_{34} = R_4^{(0)} k_4 \sin\theta_4 + R_3^{(0)} k_4 \cos\theta_4 + R_3^{(0)} \eta_4 k_4 \sin\theta_4$$
$$+R_6^{(0)} \eta_4 k_4 \cos\theta_4,$$
$$J_3 = \{R_9^{(0)} \sin\theta_3 + R_8^{(0)} \eta_3 \cos\theta_3 + R_6^{(0)} (\cos\theta_3$$
$$+\eta_3 \sin\theta_3)\}\eta_3 + R_4^{(0)} \sin\theta_3 + R_3^{(0)} (\cos\theta_3$$
$$+\eta_3 \sin\theta_3) + R_6^{(0)} \eta_3 \cos\theta_3,$$
$$J_4 = \{R_9^{(0)} \sin\theta_4 + R_8^{(0)} \eta_4 \cos\theta_4 + R_6^{(0)} (\cos\theta_4$$
$$+\eta_4 \sin\theta_4)\}\eta_4 + R_4^{(0)} \sin\theta_4 + R_3^{(0)} (\cos\theta_4 + \eta_4 \sin\theta_4)$$
$$+R_6^{(0)} \eta_4 \cos\theta_4.$$

Case II: If both the half-spaces H and H' reduce to homogeneous isotropic elastic media, the problem reduces to the reflection and refraction of elastic waves between two dissimilar isotropic elastic half-spaces. Under this condition,

$$\mu_T = \mu_L = \mu,\ \mu_T^{(0)} = \mu_L^{(0)} = \mu_0,$$
$$\alpha = \beta = \alpha^{(0)} = \beta^{(0)} = 0,$$

and the phase velocities of *qP* and *qSV*-waves are respectively are given by

$$c_1^2 = \frac{2\mu + \lambda}{\rho},\ \ c_3^2 = \frac{2\mu_0 + \lambda^{(0)}}{\rho_0},$$
$$c_2^2 = \frac{\mu}{\rho},\ \ c_4^2 = \frac{\mu_0}{\rho_0},$$

which are the results of classical elasticity (Achenbach, 1976).

The reflection and refraction coefficients and energy ratios of the reflected and refracted *qP* and *qSV*-waves are given by Eqs. (21)-(22) and (25) respectively with

$$b_{21} = -\lambda k_0 \sin\theta_0 + (\lambda + 2\mu)\eta_1 k_1 \cos\theta_1,$$
$$b_{22} = -\lambda k_0 \sin\theta_0 + (\lambda + 2\mu)\eta_2 k_2 \cos\theta_2,$$
$$b_{23} = \lambda^{(0)} k_0 \sin\theta_0 + (\lambda^{(0)} + 2\mu_0)\eta_3 k_3 \cos\theta_3,$$
$$b_{24} = \lambda^{(0)} k_0 \sin\theta_0 + (\lambda^{(0)} + 2\mu_0)\eta_4 k_4 \cos\theta_4,$$
$$b_{31} = \mu(k_1 \cos\theta_1 - \eta_1 k_0 \sin\theta_0),$$
$$b_{32} = \mu(k_2 \cos\theta_2 - \eta_2 k_0 \sin\theta_0),$$
$$b_{33} = \mu_0(k_3 \cos\theta_3 + \eta_3 k_0 \sin\theta_0),$$
$$b_{34} = \mu_0(k_4 \cos\theta_4 + \eta_4 k_0 \sin\theta_0),$$
$$\Delta = -\lambda k_0 \sin\theta_0 - (\lambda + 2\mu)\eta_0 k_0 \cos\theta_0,$$
$$\Delta' = -\mu(k_0 \cos\theta_0 + \eta_0 k_0 \sin\theta_0)$$
$$\eta_0 = \frac{-(\lambda+\mu)\sin\theta_0 \cos\theta_0}{\mu \sin^2\theta_0 + (\lambda+2\mu)\cos^2\theta_0 - \rho c_0^2},$$
$$\eta_j = \frac{(\lambda+\mu)\sin\theta_j \cos\theta_j}{\mu \sin^2\theta_j + (\lambda+2\mu)\cos^2\theta_j - \rho c_j^2},\ (j = 1, 2)$$
$$\eta_j = \frac{-(\lambda^{(0)}+\mu_0)\sin\theta_j \cos\theta_j}{\mu_0 \sin^2\theta_j + (\lambda^{(0)}+2\mu_0)\cos^2\theta_j - \rho c_j^2},\ (j = 3, 4)$$
$$R_2 = R_4 = R_6 = R_7 = R_2^{(0)} = R_4^{(0)} = R_6^{(0)} = R_7^{(0)} = m_1 = 0,$$
$$q_1 = -\lambda, R_1 = \lambda + 2\mu = R_8,\ R_3 = \mu, R_5 = \lambda + \mu,$$
$$R_1^{(0)} = \lambda^{(0)} + 2\mu_0 = R_8^{(0)},$$
$$R_3^{(0)} = \mu_0, R_5^{(0)} = \lambda^{(0)} + \mu_0,\ R_9^{(0)} = \lambda^{(0)}.$$

These results are similar with the results of the classical elasticity (Achenbach, 1976).

NUMERICAL RESULTS AND DISCUSSION

For the numerical computation of the reflection and refraction coefficients and energy ratios of the reflected and refracted waves for the incident *qP* and *qSV*-waves, we take the following relevant parameters.

For the homogeneous half-space, H :

$\lambda = 6.92\times10^9\ N/m^2$, $\mu_T = 3.05\times10^9\ N/m^2$,

$\mu_L = 4.07\times10^9\ N/m^2$, $\alpha = -0.49\times10^9\ N/m^2$,

$\beta = 0.23\times10^9\ N/m^2$, $\rho_0 = 1260 Kg/m^3$.

For the non-homogeneous half space, H' :

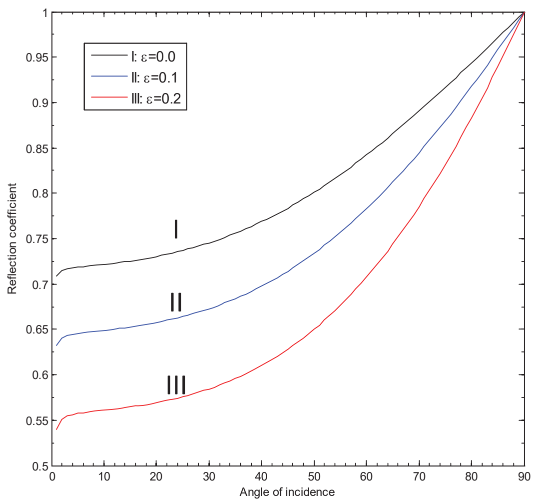
$\lambda^{(0)} = 7.59\times10^9\ N/m^2$, $\mu_T^{(0)} = 1.89\times10^9\ N/m^2$,

$\mu_L^{(0)} = 2.45\times10^9\ N/m^2$, $\alpha^{(0)} = -1.28\times10^9\ N/m^2$,

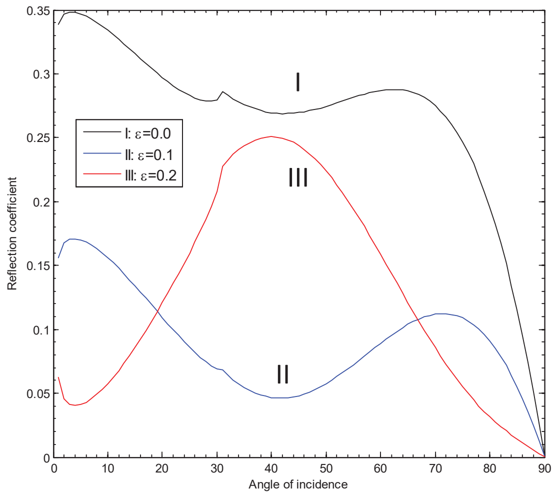
$\beta^{(0)} = 0.32\times10^9\ N/m^2$, $\rho_0 = 7800 Kg/m^3$.

The variation of the modulus of reflection and refraction coefficients with angle of incidence, θ_0 are depicted in Figures 1–4, while those of energy ratios with the angle of incidence are shown in Figures 4–8 for different values of inhomogeneity parameter, ε. In all these figures, (a) will correspond for the incident *qP*-wave and (b) will correspond for the incident *qSV*-wave.

In Figure 1(a), the modulus of the reflection coefficient, r_{pp} corresponding to the reflected *qP*-wave for incident *qP*-wave starts from certain values and increases with the increase of θ_0, attaining the maximum value at the grazing angle of incidence.



(a) Reflection Coefficient, r_{pp}

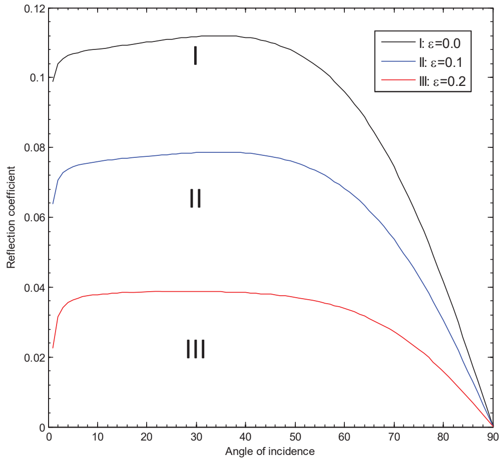


(b) Reflection Coefficient, r_{sp}

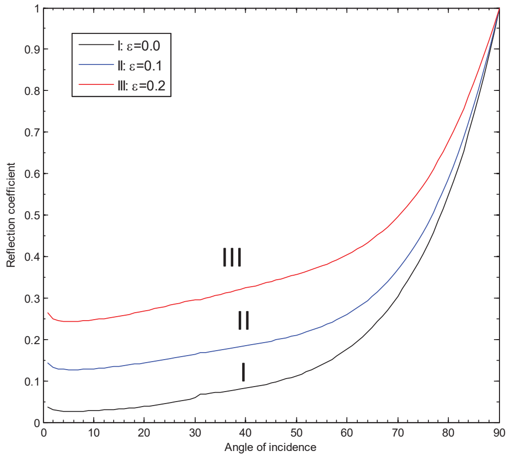
Fig. 1: Variation of the Modulus of Reflection Coefficients with Angle of Incidence, θ_0

We observed that the values of r_{pp} decrease with the increase of ε. Figure 1(b) shows the variation of the modulus of reflection coefficient, r_{sp} corresponding to the reflected *qP*-wave for the incident *qSV*-wave. Curves I and II shows that r_{sp} start from certain value at the normal incidence, which increase initially and then decrease upto $\theta_0 = 39^0$ which again increase and then decrease to zero value with the changes of θ_0. Curve III shows that r_{sp} decreases initially and increases to the maximum value at $\theta_0 = 40^0$ which decreases to zero at the grazing angle of incidence. We have seen that the minimum value of r_{sp} is at the grazing angle of incidence.

In Figure 2(a), the reflection coefficient, r_{ps} corresponding to the reflected *qSV*-wave for incident *qP*-wave increases upto $\theta_0 = 19^0$, which decreases with the increase of θ_0. The values of r_{ps} decrease with the increase of ε and it attains the minimum value at the grazing angle of incidence. The value of reflection coefficient, r_{ss} for the incident *qSV*-wave increases with the increase of θ_0 in Figure 2(b) and it attains the maximum value at the grazing angle of incidence. We observed that the values of r_{ss} increases with the increase of ε. In Figure 3, the refraction coefficients, t_{pp} and t_{sp} increase to the maximum value with the increase of θ_0 which decrease to the minimum value at the grazing angle of incidence. These coefficients increase with the increase of ε. Figure 4 shows that the refraction coefficients, t_{ps} and t_{ss} start from certain value and decrease to minimum value with the increase of θ_0. We observed that these coefficients increase with increase of ε.

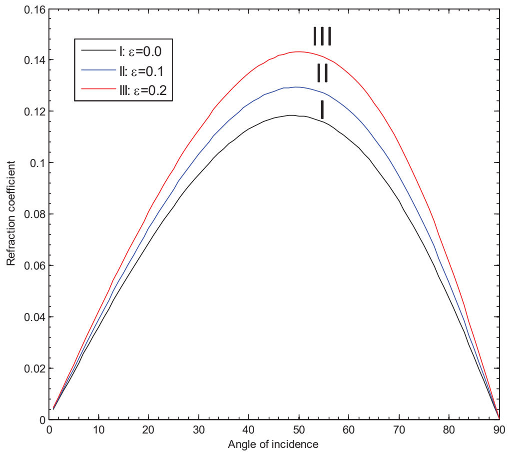


(a) Reflection Coefficient, r_{ps}

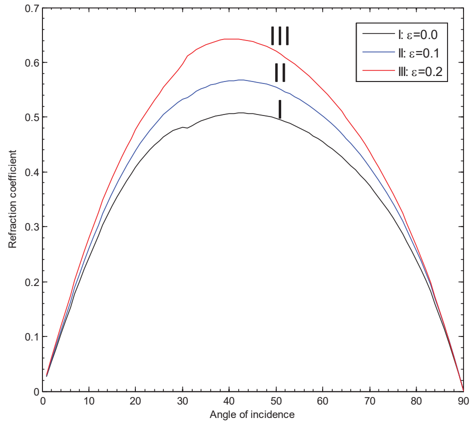


(b) Reflection Coefficient, r_{ss}

Fig. 2: Variation of the Modulus of Reflection Coefficients with Angle of Incidence, θ_0

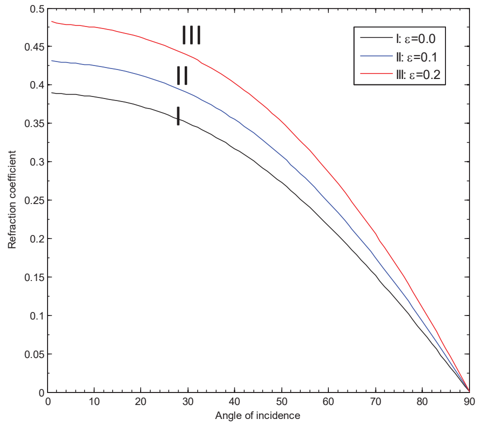


(a) Refraction Coefficient, t_{pp}

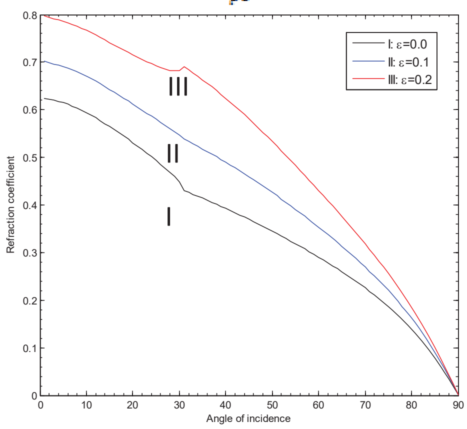


(b)Refraction Coefficient, t_{sp}

Fig. 3: Variation of the Modulus of Refraction Coefficients with Angle of Incidence, θ_0

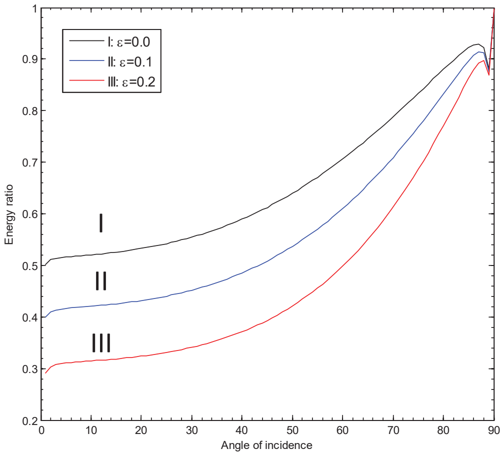


(a) Refraction Coefficient, t_{ps}

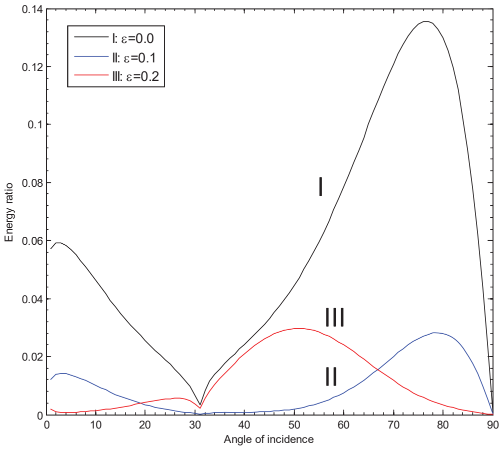


(b) Refraction Coefficient, t_{ss}

Fig. 4: Variation of the Modulus of Refraction Coefficients with Angle of Incidence, θ_0

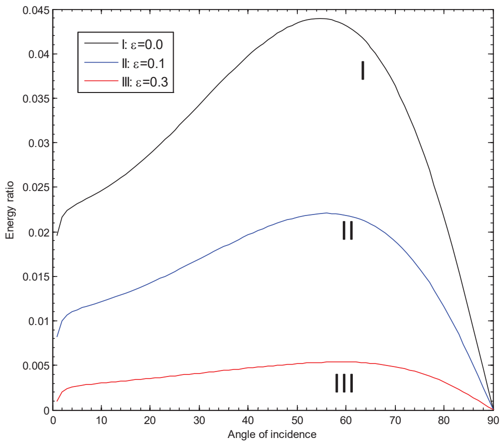


(a) Incident *qP*-waves

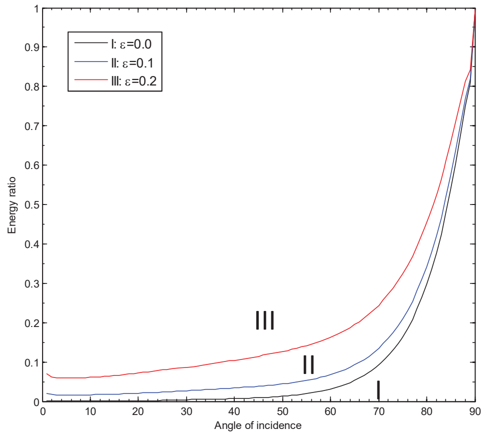


(b) Incident *qSV*-waves

Fig. 5: Variation of the Modulus of Energy Ratio, E_1 with Angle of Incidence, θ_0

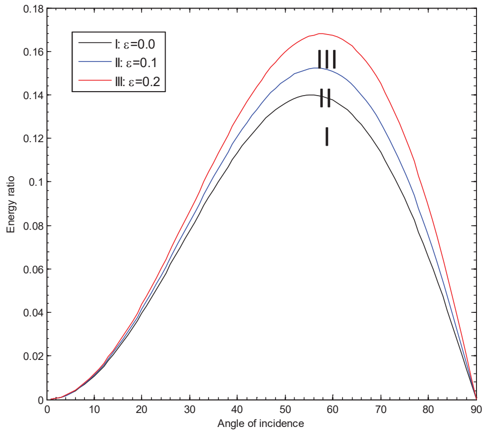


(a) Incident qP waves

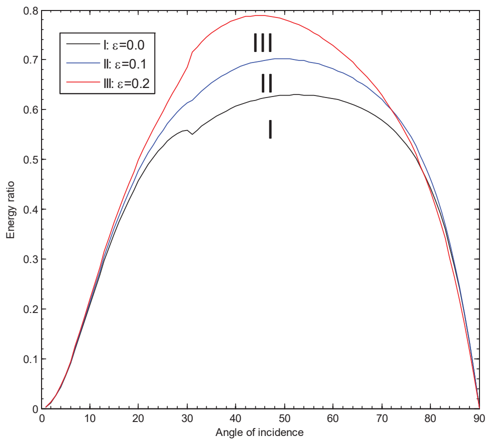


(b) Incident qSV waves

Fig. 6: Variation of the Modulus of Energy Ratio, E_2 with Angle of Incidence, θ_0

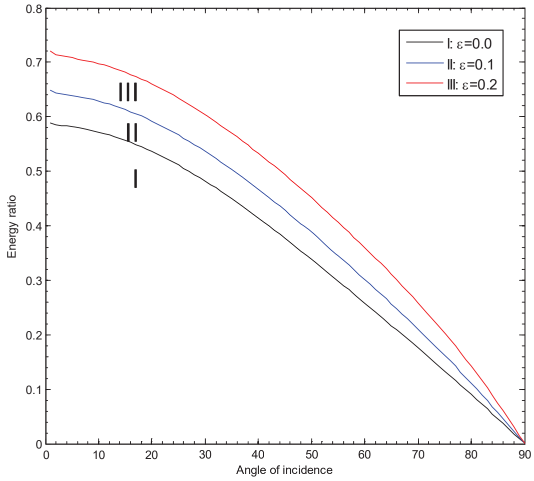


(a) Incident qP Waves

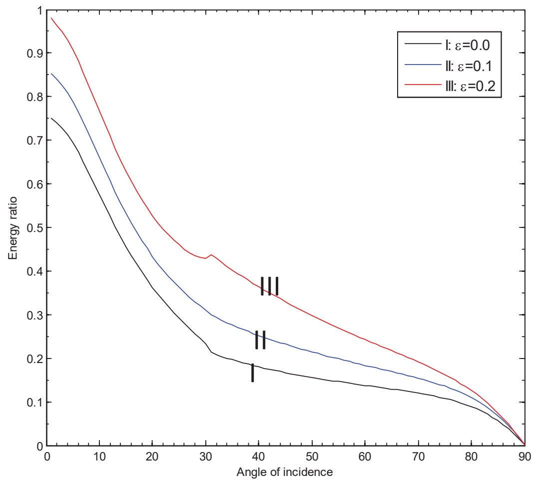


(b) Incident qSV Waves

Fig. 7: Variation of the Modulus of Energy Ratio, E_3 with Angle of Incidence, θ_0



(a) Incident qP waves



(b) Incident qSV waves

Fig. 8: Variation of the Modulus of Energy Ratio, E_4 with Angle of Incidence, θ_0

In Figure 5(a), the energy ratio, E_1 corresponding to the reflected *qP*-wave for incident *qP*-wave starts from certain values and increases upto $\theta_0 = 87^0$ and decreases thereafter with the increase of θ_0. The values of E_1 for incident *qP*-wave decrease with the increase of ε. Figure 5(b) shows the variation of the modulus of E_1 for incident *qSV*-wave with the angle of incidence. Curves I and II show that E_1 start from certain value which decreases upto $\theta_0 = 31^0$ and then increase to the maximum values at $\theta_0 =$ for Curve I and $\theta_0 =$ for Curve II, which decrease thereafter with the increase of θ_0. Curve III shows that E_1 increases initially upto certain value of θ_0 which decreases for some values and increases, thereafter to the maximumvalue, which again decreases with the increase of θ_0.

In Figure 6(a), the energy ratio, E_2 corresponding to the reflected *qSV*-wave for incident *qP*-wave increases initially with the increase of θ_0 and attains a maximum value, thereafter it decreases to zero value at the grazing angle of incidence. The values of E_2 for incident *qP*-wave decrease with the increase of ε. But, the value of E_2 for incident *qSV*-wave increases with the increase of θ_0 in Figure 6(b) and their values increase with the increase of ε. In Figure 7, the energy ratio E_3 for the incident *qP* and *qSV*-waves make a parabolic region in $0^0 \le \theta_0 \le 90^0$ and their values increase with the increase of ε. Figure 8 shows that E_4 for the incident *qP* and *qSV*-waves decrease with the increase of θ_0 and their values increase with the increase of ε. We have observed that the sum of energy ratios is close to unity.

CONCLUSION

The problem of reflection and refraction of elastic waves due to incident *qP* and *qSV*-waves at a plane interface between the homogeneous and inhomogeneous fibre-reinforced elastic half-spaces has been investigated. The reflection and refraction coefficients and energy ratios corresponding to the reflected and refracted *qP* and *qSV*-waves for the incident *qP* and *qSV*-waves are obtained analytically and numerically. We may conclude with the following points:

1. The reflection and refraction coefficients and energy ratios corresponding to the reflected and refracted waves for incident *qP* and *qSV*-waves are functions of elastic constants, fibre orientation, inhomogeneity parameter and angle of incidence.
2. The energy ratios, E_1 for incident *qP*-wave, E_2 for incident *qSV*-wave and reflection coefficients, r_{pp}, r_{ss} attain the maximum value at grazing angle of incidence.
3. The energy ratio E_2 for incident *qSV*-wave and the coefficients, r_{pp}, r_{ss} increase with the increase of angle of incidence.
4. The energy ratio, E_4 and the coefficients, t_{ps}, t_{ss} decrease with the increase of θ_0.
5. The values of energy ratios, E_3, E_4 and the coefficients, r_{ss}, t_{pp}, t_{sp}, t_{ps}, t_{ss} increase with the increase of inhomogeneity parameter, ε.
6. The sum of energy ratios is close to unity.

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REFERENCES

Abbas IA (1972) A two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation. Sadhana 36: 411–423.

Abbas IA, Abd-alla AN (2011) A study of Generalized Thermoelastic Interaction in an infinite fibre-reinforced anisotropic plate containing a circular hole. Acta Phys. Pol. A 119: 814–818.

Achenbach JD (1976) Wave propagation in elastic solids. North-Holland Publishing Company, New York.

Bedford A, Sutherland HJ (1973) Reflection and transmission of waves by a fiber-reinforced material. J. Acoust. Soc. Am. 54: 1273–1276.

Belfield AJ, Rogers TG, Spencer AJM (1983) Stress in elastic plates reinforced by fibres lying in concentric circles. J. Mech. Phy. Sol. 31: 25–54.

Cates ME, Edwards SF (1984) Linear theory of disordered fibre-reinforced composites. Proc. R. Soc. Lond. A 395: 89–109.

Chattopadhyay A, Singh AK (2012) *G*-type seismic waves in fibre reinforced media. Meccanica 47: 1775–1785.

Chattopadhyay A, Venkateswarlu RLK (2007) Reflection and refraction of quasi *P* and *SV* waves at the interface of fibre-reinforced media. Adv. Studies Theor. Phys. 1: 57–73.

Chattopadhyay A, Venkateswarlu RLK, Saha, S (2002) Reflection of quasi-*P* and quasi-*SV* waves at the free and rigid boundaries of a fibre-reinforced medium. Sadhana 27:613–630.

Chen TJ, Chen CS, Chen CW (2011) Dynamic response of fiber-reinforced composite plates. Mech. Comp. Mat. 47: 549–560.

Green WA (1991) Reflection and transmission phenomenon of transient stress waves in fibre composite laminates. Rev. Prog. Quant. NDE 10: 1407–1414.

Othman MIA, Lotfy K, Said SM, Beg OA (2012) Wave propagation in a fiber-reinforced micropolar thermoelastic medium with voids using three models. Int. J. Appl. Math. Mech. 8: 52–69.

Singh B (2006) Wave propagation in thermally conducting linear fibre-reinforced composite materials. Arch. Appl. Mech. 12: 1–7.

Singh B (2007) Wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. Arch. Appl. Mech. 77: 253–258.

Singh B, Singh, SJ (2004) Reflection of plane waves at the free surface of a fibre-reinforced elastic half-space. Sadhana 29: 249–257.

Singh SS, Tomar SK (2007a) Shear waves at a corrugated interface between two dissimilar fiber-reinforced elastic half-spaces. J. Mech. Mater. Struct. 2: 167–188.

Singh SS, Tomar SK (2007b) Elastic waves at a corrugated interface between two dissimilar fibre-reinforced elastic half-spaces. Int. J. Num. Analy. Methods. Geomech. 31: 1085–1116.

Singh SS, Zorammuana C (2014) Incident longitudinal wave at a fibre-reinforced thermoelastic half-space. J. Vib. Cont. 20: 1895–1906.

Zorammuana C, Singh SS (2015a) Elastic waves in thermoelastic saturated porous medium. Meccanica 51: 593–609.

Zorammuana C, Singh SS (2015b) *SH*-wave at a plane interface between homogeneous and inhomogeneous fibre-reinforced elastic half-spaces. Indian J. Mater. Sc. Article ID 532939: 1–8.

Spencer AJM (1972) Deformation of fibre-reinforced materials, Clarendon Press, Oxford.

Tomar SK, Singh SS (2006) Plane *SH*-waves at a corrugated interface between two dissimilar perfectly conducting self-reinforced elastic half-spaces. Int. J. Num. Anal. Met. Geomech. 30: 455–487.