(b) Define planar graph. What are the properties of a planar graph? 2+3=5

(c) Prove that k_5 and $k_{3,3}$ are non-planar. 4

10. (a) Explain why the Konigsberg bridge problem has no solution. 2

(b) Give one example of a simple graph which has Hamiltonian path but no Hamiltonian cycle. 1

(c) Prove that Kuratowski's graphs are non-planar. 6

(d) State and derive the Euler's formula for plane graph. 1+4=5

★★★

PG (CBCS) EVEN SEMESTER EXAMINATION, 2023

MATHEMATICS

2nd Semester

Course No. : MTMCC - 204

(Discrete Mathematics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

All the symbols have their usual meanings

(Answer one question from each unit)

Unit - I

1. (a) If n is an odd integer show that $n^4 + 4n^2 + 11$ is of the farm 16k for same integer k. 4

(b) Use Eucledian algarithm to obtain intergers x and y satisfying gcd (24, 138) = $24x + 138y$ 5

(c) Prove that the product of four consecutive integer is 1 less than perfect square. 5

2. (a) Prove that if gcd (a, b) = 1, then gcd (a + b, ab)=1 3

(b) Determine all the solutions in positive integers of the following diophantine equation
$11x + 5y = 79$ 6

(Turn Over)

(c) The circumference of a Ralette whell is divided into 36 sectors to which the numbers 1,2, ..., 36 are assigned in some arbitrary manner. Show that there are three consecutive sectors such that the sum of their assigned number is at least 56 5

Unit - II

3. (a) Prove that, if p is a prime, then $(p - 1)! \equiv -1 \pmod p$ 5

(b) Find all the prime numbers that divide 50! 4

(c) If $a \equiv b \pmod{p^k}$, prove that $a^p \equiv b^p \pmod{p^{k+1}}$ 5

4. (a) Show that 5 is a primitive root of 18 4

(b) State and prove Fermat's little theorem. 5

(c) Prove that the integer $53^{103} + 103^{53}$ is divisible by 39 that $111^{333} + 333^{111}$ is divisible by 7. 5

Unit - III

5. (a) Solve $3y^2 + 2y + 3 \equiv 0 \pmod{11}$ by using quadratic reciprocity law. 5

(b) Prove that the product of a quadratic residue and a quadratic non-residue of p is a non-quadratic residue at p. 4

(c) Find all the primitive roots of 19. 5

6. (a) Find the solutions of $2^{15} \equiv 7 \pmod{19}$ 6

(b) State and prove Möbious inversion formula. 2+6=8

Unit - IV

7. (a) Let G be a graph having distinct vertices *u* and *v*. If G contains *u*-*v* walk then prove that G contains a *u*-*v* path. 6

(b) Define k-regular graph and show that k_n has exactly $\frac{n(n-1)}{2}$ edges. 5

(c) Prove that any graph G with atleast six vertices, G or $\overline{G}$ contains a triangle. 3

8. (a) Prove that every tree with atleast two vertices has at least two leaves. 6

(b) If G is a tree with p vertices and q edges, show that every two vertices of G are joined by a unique path and hence $p = q + 1$ 6

(c) Define complete bipartile graph. Draw the graph $k_{2,5}$. 2

Unit - V

9. (a) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. 5

(Turn Over)