(c) Obtain canonical solution of one-dimensional wave equation by canonical reduction. 4

8. (a) Find the general solution of Euler-Poisson-Darboux equation. 7

(b) Solve the following initial value problem of the wave equation defined by $u_{tt} - c^2 u_{xx} = f(x, t)$

Subject to

$u(x, 0) = \eta(x), \; u_t(x, 0) = v(x)$ 7

<u>Unit - V</u>

9. (a) Find the general and most acceptable solution of $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ 7

(b) Prove that if a Dirichlet problem for bounded region has a solution, then it is unique. 7

10. (a) Solve $\nabla^2 u = 0, \; 0 \le x \le a, \; 0 \le y \le b$ satisfying $u(0, y) = 0, \; u(x, 0) = 0, \; u(x, b) = 0,$ 7

$$\frac{\partial u}{\partial x}(a, y) = T \sin^3\left(\frac{\pi y}{a}\right)$$

(b) Find potential function u (x, y, z) in rectangular box defined by $0 \le x \le a, \; 0 \le y \le b, \; 0 \le z \le c$; if the potential is zero on all the sides and the bottom while $u = f(x, y)$ on the top of the box. 7

★★★

PG (CBCS) EVEN SEMESTER EXAMINATION, 2023

MATHEMATICS

2nd Semester

Course No. : MTMCC - 205

(Partial Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

<u>Unit - I</u>

1. (a) Eliminate the arbitary function F from

(i) $F(z - xy, x^2 + y^2) = 0$ (ii) $F(xy, x + y - z) = 0$

and find corresponding partial differential equations. 5

(b) Find the complete integral of $f = z^2 - pqxy = 0$ 4

(c) Find the general solution of 3+3=6

(i) $y^2 p - xyq = x(z - 2y)$

(ii) $yzp + xzq = x + y$

2. (a) Prove the necessary and sufficient condition of integrability of

(Turn Over)

$dz = \phi (x, y, z)\, dx + \psi (x, y, z)\, dy$ is 5

$$\frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$$

(b) Solve : 5

$z + 2u_z - (u_x + u_y)^2 = 0$ by Jacobi's method.

(c) Find integral surface of $pq = z$ containing the curve $x = 0$, $z = y^2$ 4

Unit - II

3. (a) Find the families of characteristics of $(1 - x^2)\, u_{xx} - u_{yy} = 0$ in elliptic and hyperbolic cases. 3+3=6

(b) Obtain the canonical form of $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ when $S^2 - 4RT > 0$ 8

4. (a) Reduce $t - s + p - q\left(1 + \frac{1}{x}\right) + \left(\frac{z}{x}\right) = 0$ to caronical form. 5

(b) Classify : 5

(i) $2u_{xx} + 4u_{xy} + 3u_{yy} = 2$

(ii) $u_{xx} + 4u_{xy} + 4u_{yy} = 0$

(iii) $(x - y)(xr - xs - ys + yt) = (x + y)(p - q)$

(c) Find characteristics of $y^2 r - x^2 t = 0$ 4

Unit - III

5. (a) Find all possible solutions of 5

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(b) A one dimensional infinite region $-\infty < x < \infty$ is initially kept at zero temperature. A heat source of strength g_s unit, situated at $x = \xi$ releases its heat instantaneously at time $t = \tau$. Determine the temperature in the region for $t > \tau$. 5

(c) Find the temperature in a sphere of radius a when its surface is kept at zero temperature and its initial temperatures $f(r, \theta)$ 4

6. (a) Obtain solution of diffusion equation in cylindrical co-ordinates. 7

(b) In a one-dimensional infinite solid, $-\infty < x < \infty$, the surface $a < x < b$ is initially maintained at temperature to and at zero temperature everywhere outside the surface. Show that

$$T(x,t) = \frac{T_0}{2} = \left[\operatorname{erf}\left(\frac{b - x}{\sqrt{4\alpha t}}\right) - \operatorname{erf}\left(\frac{a - x}{\sqrt{4\alpha t}}\right)\right]$$ 7

Unit - IV

7. (a) Find solution of $\nabla^2 u + k^2 u = 0$ 4

(b) State and prove uniqueness theorem for the solution of wave equation. 6