PG (CBCS) EVEN SEMESTER EXAMINATION, 2023

MATHEMATICS

4th Semester

Course No. : MTMCC - 403

(Fluid Dynamics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

Unit - I

1. (a) (i) What are different ways of describing the fluid motion? Discuss them in comparative manner. 1+3=4

(ii) Define stream lines and path lines of flow. Write the condition for which they represent the same line. 2+1=3

(b) Write the physical interpritation of conservation of mass principle in regard to a flow. Hence, determine the constants k_1, k_2 and k_3 in order that the velocity profile,

$$\vec{q} = \frac{1}{r(x+r)} \ (x + k_1 r \, , y + k_2 r \, , z + k_3 r)$$

where $\vec{r} = (x, y, z)$ may satisfy the law of

conservation of mass, if density (e) = constant. 2+5=7

2. (a) Derive Lamb's hydrodynamical equations for flow of perfect fluid under rotational effect. Can the Euler's equations be derivable from it? If so, how? 4+2=6

(b) What is Kelvin's circulatio theorem? How it can be used to justify the performance of irrotational motion? If a flow is characterized by $\vec{q} = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}\right)$ find the circulation round a rectangle { (0, 1), (3, 1), (3, 2), (0, 2)}. Hence, interpret your result. 1+2+5=8

Unit - II

3. (a) What is current function? Discuss it's physical interpretation. If the current function of a 2-D flow is given by ψ (x, y, t) = x (t + y) ; find the velocity profile. Also, obtain the velocity potential function, if possible. 1+2+4=7

(b) Show that the force perunit length exerted on a circular cylinder of radius 'R' due to a source of strength '+m' situated at a distance 'd' from the axis of the cylinder is $\frac{2\pi e m^2 R^2}{d\,(d^2 - R^2)^2}$, where d>R. 7

Unit - V

9. (a) What is boundary layer theory of Prandil? Obtain the boundary layer equations from the Navier-stoke's equations using the order of magnitude analysis. 2+5=7

(b) Calculate a cubic polynomial form for velocity distribution in a boundary layer and hence find the boundary layer thickness. 4+3=7

10. (a) Deduce the Blasins equation from the 2-D boundary layer equatios using a suitable transformations. 7

(b) Find the boundary layer thickness, co-efficient of skin-friction for the velocity distribution in a boundary layer as :

$$\frac{u}{v_\infty} = a + b\eta + c\eta^3$$

where, $\eta = \frac{y}{\delta}$ as the similarity variable defined in the boundary layer. 7

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4. (a) What are 2-D sources, sinks and doublets? Sources of equal strength are placed at the points Z = ika, where k ∈ I (set of integers). Prove that the complex potential function associated with the flow in presence of the source is $-mln\left\{\sinh\left(\frac{\pi z}{a}\right)\right\}$. Hence, obtain the stream lines of flow. 3+4+3=10

(b) Suppose, a uniform stream of flow U_0 flowing past a circular cylinder, construct the complex potential function, using the circle theorem. 5

Unit - III

5. (a) Discuss the motion arround a sphere, moving through an infinite mass of a liquid at rest at infinity, if a velocity U_0 is additionally injected to the sphere to speed up its motion. 8

(b) An infinite row of equidistant rectilinear vertices of equal numerical strength 'k', but alternately of opposite signs are place at distances 'd' apart in an infinite liquid. Obtain the speed of fluid particles due to the presence of vertices. 6

6. (a) Suppose, a liquid slides over a spherical object of radius 'R' with velocity U_0; establish that the velocity of the fluid particles on the top of the

object at any instant of time is half the velocity of the spherical object. 5

(b) Under Von-Karman's vortex street model, obtain the pressure at a point due to the vortex system. 9

Unit - IV

7. (a) What is Hagen-Poisecille flow? A viscous liquid flows steadily and parallel to the axis in the annular space between two co-axial cylinders of radii 'a' and 'ka' (k>1) respectively. Prove that the rate of discharge of flow is $\frac{\pi p a^4}{8\mu}\left(K^4 - \frac{(k^2-1)^2}{lnk} - 1\right)$ where 'p' is the constant pressure gradient, 'μ' is the dynamic viscosity. 7

(b) An unsteady flow is developed due to sudden movement of a flat plate along its length. Obtain the velocity of the flow. 7

8. (a) A steady flow takes place through a parallel channel consists of two flat plates moving relatively to one another; calculate the rate of flow discharge per unit time. 6

(b) An internal steady flow occurs through a pipe of elliptic cross-section; find the velocity profile of the flow. 8