8. (a) State and prove Euler's theorem. 6

(b) Show that the Dupin indicatrix at every point on the right helicoid 5
r = (u cos v, u sin v, c v) is a rectangular hyperbola, where c is a constant.

(c) Show that $e^z \cos x = \cos y$ is minimal surface. 3

Unit - V

9. (a) When v = costant for all values of u, prove that a necessary and sufficient condition that the curve v = constant is a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ 8

(b) Prove that every helix on a cylinder is a geodegic and conversely. 6

10. (a) Show that the curves u+ v= constant are geodesic on a surface with the metric 8
$(1 + u^2)\, du^2 - 2uv\, du\, dv + (1 + v^2)\, d v^2$

(b) Show that a curve on a sphere is a geodesic if and only if it is a great circle. 6

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PG (CBCS) EVEN SEMESTER EXAMINATION, 2023

MATHEMATICS

4th Semester

Course No. : MTMCC - 404(B)

(Differential Geometry)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(All the symbols have their usual meanings)

(Answer one question from each unit)

Unit - I

1. (a) If R = R(u) is the parametric representation of a curve where u ∈ [a, b] then prove that the length of the curve is 6

$$\int_a^u \left|\dot{R}(u)\right| du$$

(b) State and prove the necessary and sufficient condition for a curve to be a straight line. 4

(c) Find the curvature and torsion of the curve r = (cos 2u, sin 2u, 2 sin u) 4

2. (a) If the radius of the spherical curvature is constant, prove that curve either lies on a sphere or has a constant curvature. 5

(b) Derive the formula to find the curvature and torsion of spherical indicatrix of the tangent to a curve. 4

(c) State and prove the necessary and sufficient condition for a curve to be a helix. 5

Unit - II

3. (a) Obtain the surface equation of sphere and find the singularities, parametric curves, tangent plane at a point and the surface normal. 9

(b) Show that on the surface r = (a (u+v), b (u-v), uv) the parametric curves are straight lines. 5

4. (a) Prove that the tangents to the edge of regression are the characteristic lines of the developable. 5

(b) Find the equation of the tangent plane to the surface $z = xy$ at (2, 3, 6) 5

(c) Examine the nature of the parametric curves on the surface 4

r = (u cos v, u sin v, u v)

Unit - III

5. (a) Prove that the first fundamental form is invarient under parametric transformation. 5

(b) Calculate the first fundamental co-efficients and the area of the anchor ring. 5

r = ((b + a cos u) cos v, (b + a cos u) sin v, a sin u)

(c) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the section by the planes z = constant. 4

6. (a) If $v^2du^2 + u^2 dv^2$ is the metric on a surface, find the family of curves orthogonal to the curve $\frac{u}{v}$ = constant and find the metric referred to the new parameters so that these two families are parametric curves. 7

(b) Find the normal curvature of the right helicoid r (u, v) = (u cos v, u sin v, c v) 3

at a point on it. Here c is a constant.

(c) State and prove Meusiner's theorem. 1+3=4

Unit - IV

7. (a) Derive the equation to find the principal curvature. 6

(b) Find the Gaussian curvature of r (u, v) = (u cos v, u sin v, cos 2 v) 3

(c) Show that the parametric curves on the surface. r = (u cos v, u sin v, c v) are asymptotic lines, where c is a constant. 5