# PG EVEN SEMESTER EXAMINATION, 2023

## MATHEMATICS

2nd Semester

Course No. : MAT - 553

**( Partial Differential Equations )**

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

(Answer one question from each unit)

Unit - I

1. (a) Show that $2z = (ax + y)^2 + b$ is a complete integral of $px + qy - q^2 = 0$. 4

   (b) Verify that $(y^2 + yz)\, dx + (xz + z^2)\, dy + (y^2 - xy)\, dz = 0$ is integrable and find the corresponding integral. 5

   (c) Show that $f \equiv xp - yq - x = 0$ & $g \equiv x^2p + q - xz = 0$ are compatible of common solutions. 5

2. (a) Find the complete integral of $p^2q^2 + x^2y^2 = x^2q^2\,(x^2+y^2)$ using Charpit method. 5

( Turn Over )

(b) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$, which passes through the line $x_0(s) = 1$, $y_0(s) = 0$ and $z_0(s) = 8$ 5

(c) Find the general solution of $yzp + xzq = xy$ 4

Unit - II

3. (a) Reduce

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form and hence solve it. 5

(b) Find the characteristics of $(\sin^2 x)\, r + (2\cos x)\, s - t = 0$ 5

(c) Find the surface satisfying $t = 6x^2 y$ containing two lines $y = 0 = z$, $y = 2 = z$ 4

4. (a) Solve $t + s + q = 0$ by Lagrange's method. 5

(b) Solve $t = \sin xy$

(c) Find the surface passing through the circle $x^2 + y^2 = a^2$, $z = 0$ and satisfying the differential equation $s = 8xy$ 4

Unit - III

5. (a) Obtain the periodic solution of the wave equation in the form $u(x, t) = Ae^{i(kx + wt)}$ where $i = \sqrt{-1}$, $k = \pm w/c$, A is constant and hence define various terms involves in wave propagation. 4



(c) State and prove stability theorem. 4

Unit - V

9. (a) Apply seperation of variables method to solve $u_{xx} + u_{yy} = 0$ 5

(b) Find the potential function u (x, y, z) in a rectangular box defined by $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$, if the potential is zero on all sides and the bottom, while $u = f(x, y)$ on the top of the box. 5

(c) Solve $\nabla^2 u = 0$, $0 \le x \le a$, $0 \le y \le b$ satisfying boundary conditions $u(0, y) = 0$, $u(x, 0) = 0$, $u(x, 4) = 0$ 4

$$\frac{\partial u}{\partial x}(a, y) = T \sin^3 \frac{\pi y}{a}$$

10. (a) Find the solution of 4

$\nabla^2 u + K^2 u = 0$

(b) Find the potential at all points of space in side and outside of a sphere of radius R = 1 which is maintained at a constant distribution of electric potential $u(R, \theta) = f(\theta) = \cos 2\theta$ 5

(c) Find the general spherically symmetric solution of $(\nabla^2 - K^2)\, u = 0$ 5

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Find the resultant time dependent motion of the string. 5

## Unit - IV

7. (a) Show that $T(x,t) = \dfrac{1}{\sqrt{4\pi\alpha t}} \exp\left[\dfrac{-(x-\xi)^2}{4\alpha t}\right]$

is elementary solution 4

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, -\infty < x < \infty, \qquad t > 0$$

(b) Solve one-dimensional diffusion equation in the region $0 \leq x \leq \pi$, $t > 0$, subject to the conditions that

(i) T remains finite as $t \to \propto$

(ii) $T = 0$ if $x = 0$ and $\pi$ for all t

(iii) At $t = 0$, $T = \begin{cases} x, & 0 \leq x \leq \pi/2 \\ \pi - x, & \pi/2 \leq x \leq \pi \end{cases}$ 5

(c) Obtain the solution of diffusion equation in cylindrical co-ordinates. 5

8. (a) Find the temperature in a sphere of radius a when its surface is kept at zero temperature and its initial temperature is $f(r, \theta)$ 5

(b) Deermine the temperature $T(r, t)$ in the infinite cylinder $0 \leq r \leq a$ when the initial temperature is $T(r, 0) = f(r)$ and the surface $r = a$ is maintained at $0^0$ temperature. 5



(b) Solve the following 5

$u_{tt} - c^2 u_{xx} = 0$, $-\propto < x < \propto$, $t > 0$

subject to initial conditions

$u(x, 0) = \eta(x)$, $u_t(x, 0) = v(x)$ 5

(c) Show that solution of $u_{tt} = c^2 u_{xx}$. $0 < x < L$, $t>0$ satisfying initial condition

$u(x, 0) = f(x)$, $0 < x < L$

$u_t(x, 0) = g(x)$, $0 < x < L$

and boundary conditions

$u(0, t) = u(L, t) = 0$

where $u(x, t)$ is twice continuously differentiate function with respect to $x$ and $t$ is unique. 5

6. (a) Obtain the solution of one-dimensional wave equation by canonical reduction. 4

(b) Prove that total energy of a string, which is fixed at the points $x = 0$ and $x = 2$ and executing small transverse vibrations, is given by

$$\frac{1}{2} T \int_0^L \left[ \left(\frac{\partial y}{\partial x}\right)^2 + \frac{1}{c^2}\left(\frac{\partial y}{\partial t}\right)^2 \right] dx$$

where $c^2 = T/P$ is the uniform linear density and T is the tension. Show that if $y = f(x - ct)$, $0 \leq x \leq L$, then the energy of the wave is equally divided between potential energy and kinetic energy. 5

(c) A stretched string of finite length L is hold fixed at its ends and is subjected to an initial displacement $u(x, 0) = u_0 \sin(\pi x/L)$. The string is released from this position with zero velocity.