# PG EVEN SEMESTER EXAMINATION, 2023

## MATHEMATICS

2nd Semester

Course No. : MAT - 554

**( Introduction to Graph Theory )**

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

All the symbols have their usual meanings

(Answer one question from each unit)

Unit - I

1. (a) If G is a tree, then prove that $\epsilon = \nu - 1$ 3

(b) Construct the intersection graph of the following sets - 3

$A_1 = \{ x \mid x < 0 \}$ , $A_2 = \{ x \mid -1 < x < 0 \}$,
$A_3 = \{ x \mid 0 < x < 1 \}$ , $A_4 = \{ x \mid -1 < x < 1 \}$,
$A_5 = \{ x \mid x > -1 \}$ , $A_6 = \mathbb{R}$

(c) Let G be a graph with $\nu - 1$ edges. Prove that the following three statements are equivalent. 5

(i) G is connected (ii) G is acyclic (iii) G is a tree

(d) Prove that in any graph the number of vertices of odd degree is even. 3

( Turn Over )

2. (a) Prove that a graph is bipartite if and only if all its cycles are even. 5

(b) Show that following graphs are not isomorphic. 3

(c) Prove that every column sum of the incidence matrix is 2. 2

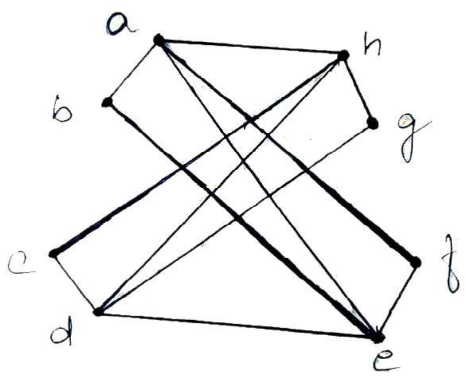
(d) Prove that an edge e of a graph G is a cut edge of G if and only if e is contained in no cycle of G. 4

## Unit - II

3. (a) Prove that a connected graph G is an Eular graph if and only if all vertices of G are of even degree. 4

(b) Prove that a graph is Eularian if and only if it is connected and if the set of its edges can be partitioned into different union of cycles. 5

(c) Obtain the Eulerian circuit from following graph using Fleury's algorithm. 5



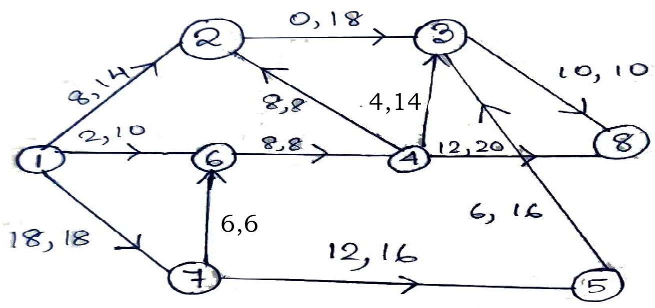



## Unit - V

9. (a) If f is any feasible flow in a capacitated network and if (S, T) is any cut in the network, then prove that f (G) = f (S, T) - f (T, S) 4

(b) State and prove Ford-Fulkerson theoem. 1+3=4

(c) Obtain a maximum flow and minimum cut in the following network. 6




10. (a) State and prove Menger's theorem of vertex form for directed graphs 1+5=6

(b) Find the minimum number of edges neded to ensure that a simple grap is connected. 4

(c) Construct the Harary graphs $H_{k,\,n}$ for

(a) n = 6 , k - 4 2

(b) n = 6 , k = 5 2

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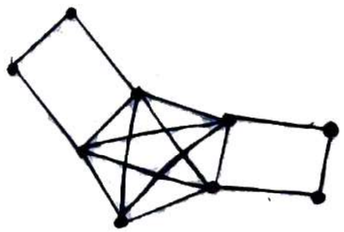
6. (a) Find the chromatic number of the following graph using Brook's theorem. 4



(b) Let G be a k-critical graph with a 2-vertex cut {u, v}, then prove that $G = G_1 \cup G_2$ where $G_i$ is a {u, v} component of type i, i = 1,2 5

(c) If G is bipartite, then prove that 5

$$\chi' (G) = \Delta (G)$$

## Unit - IV

7. (a) Define Hamiltonian graph with examples. 4

(b) Prove the following properties of the planar graph

(i) $r \leq \frac{2e}{3}$ 3+4+3=10

(ii) $r = e - \upsilon + 2$

(iii) $e \leq 3\upsilon - 6$

8. (a) Prove that $K_{3,3}$ and $K_5$ are non planar and hence state Kuratowski's theorem. 4+4+2=10

(b) Show that graph $K_4$ and the graphs $K_{2,n}$ are planar for all $n \in N$. 4



4. (a) (i) Draw a graph which has hamiltonian path but no Hamiltonian cycle. 1

(ii) Draw a graph which has no Hamiltonian path and no Hamiltonian cycle. 1

(b) State and prove Ore's theorem. 2+5=7

(c) Prove that a graph with a cut vertex is not Hamiltonian. 3

(d) Prove that a graph is Hamiltonian if and only if its closure is Hamiltonian. 2

## Unit - III

5. (a) Prove that every k-chromatic graph has at least k vertices of degree at least k-1. 3

(b) If he maximum vertex degree of a graph G is $\Delta$ then prove that $\chi (G) \leq \Delta + 1$ 3

(c) If the graph has a tree with n vertices then it has chromatic polynomial of the farm $x (x - 1)^{n-1}$. 4

(d) Let G be a connected graph that is not an odd cycle. Prove the G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two. 4