8. (a) Derive Runge-Kutta method of order 2 for solving

IVP : $\frac{dy}{dx} = f(x,y)$ such that $y(x_0) = y_0$ 4

(b) Derive the error in the approximate solution of

$$\frac{d^2y}{dx^2} + f(x).\frac{dy}{dx} + g(x).y = r(x)$$

with $y(x_0) = a$ and $y(x_n) = b$ by using Finite-Difference method of order '2'. 4

Unit - V

9. (a) Classify the following PDEs :

(i) $u_{xx} + 2.u_{xy} + u_{yy} = 0$

(ii) $u_{xx} + 4\,u_{xy} + (x^2 + 4y^2).\,u_{yy} = \sin(xy)$

(iii) $(1 + x^2)\,u_{xx} + (5 + 2x^2).\,u_{xt} + (4 + x^2).\,u_{tt} = 0$ 3

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $0 < x < 1$, $t > 0$ given that $u(0, t) = 0$, $u(1, t) = 0$ and $u(x, 0) = 100\,(x - x^2)$, by using Crank-Nicolson's method. 5

10. Solve $\frac{\partial^2 u}{\partial t^2} = 16.\frac{\partial^2 u}{\partial x^2}$ given that $u(0, t) = 0$, $u(5, t) = 0$, $u(x, 0) = x^2(x - 5)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$ 8

★★★

PG EVEN SEMESTER EXAMINATION, 2023

MATHEMATICS

2nd Semester

Course No. : MAT - 555

Numerical Analysis (Theory)

Full Marks : 40

Pass Marks : 16

Time : 2 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

Unit - I

1. (a) Let $x_0, x_1, x_2, \ldots, x_n$ be district real numbers and $y_0, y_1, y_2, \ldots, y_n$ be corresponding orbitrary real numbers. Show that there exists a unique interpolating polynomial of $P_n(x)$ of degree atmost 'n' such that $P_n(x_i) = y_i$, for $i = 0, 1, 2, \ldots, n$. 5

(b) Construct the Lagrange's interpolating polynomial of degree '2' for the function $f(x) = e^x$ on $[-1, 1]$ with interpolating points $x_0 = -1$, $x_1 = 0$, $x_2 = 1$. 3

2. (a) Suppose $n \geq 0$ and $f(x)$ is real valued function and continuous on $[a, b]$. Let the derivative of $f(x)$ of order $(n+1)$ exist and it is continuous on $[a, b]$. Show that there exists some $\xi \in (a, b)$ such that

$$|f(x) - P_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |\pi_{n+1}(x)|, \quad \text{where} \qquad 5$$

$$M_{n+1} = \max_{s \in [a,b]} |f^{n+1}(s)|$$

and $\pi_{n+1}(x) = (x-x_0)(x-x_1) \ldots (x-x_n)$

(b) Construct the piecewise cubic interpolating polynomial for the function f(x) defined by the following data :

x	- 3	- 2	- 1	1	3	6	7
y	369	222	171	165	207	990	1779

Unit - II

3. (a) Derive the 3-point derivative formula for approximating the value of f ′(x) at some nodal point by using the method of undetermined coefficients. Also, find the error in this approximation. 5

(b) Find 'n' such that the error in the approximation of the integral $\int_0^1 \cos(\pi x)\, dx$, using composite trepozoidal rule, is less than 10^{-6}. 3

4. (a) Derive the simpson's rule and then composite Simpon's rule. Also, show that Simpson's rule is exact for plynomials of degree ≤ 3. 6

(b) How large should 'n' be chosen in order to ensure that the error in composite Simpson's rule is less than 10^{-5} in the computation of $\int_0^1 e^x\, dx$ 2

Unit - III

5. (a) Show that the order of convergence of the Newton-Raphson method is 2. 5

(b) Suppose we want to compute the positive root of the equation $x = a - bx^2$; a, b > 0 by using fixed-point interation method $x_{n+1} = a - b\, x_n^2$. Derive the condition for its convergence. 5

6. (a) Let $\in_n$ and $\in_{n+1}$ be the errors at n^{th} and $(n+1)^{th}$ interations, respectively, computed in Secant method. Show that 4

$$\epsilon_{n+1} \cong C.\epsilon_n\, \epsilon_{n-1}, \qquad \text{where } C = \frac{1}{2} \cdot \frac{f''(p)}{f'(p)}$$

(b) Find the real root of the equation cox = 3x-1 correct to four decimal places using fixed-point iteration method. 4

Unit - IV

7. (a) Derive Modified Euler's method for solving

$$\text{IVP} : \frac{dy}{dx} = f(x,y)\ ;\ y(x_0) = y_0 \qquad 4$$

(b) For a given $\text{IVP} : \frac{dy}{dx} = x - y^2\ ;\ y(0) = 1$ find y (0.2) correct to four decimal places by using Taylor's series method of order 4. 4